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Multiobjective differential evolution algorithm based on decomposition for a type of multiobjective bilevel programming problems

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ABSTRACT

This paper considers the multiobjective bilevel programming problem (MOBLPP) with multiple objective functions at the upper level and a single objective function at the lower level. By adopting the Karush-Kuhn-Tucker (KKT) optimality conditions to the lower level optimization, the original multiobjective bilevel problem can be transformed into a multiobjective single-level optimization problem involving the complementarity constraints. In order to handle the complementarity constraints, an existing smoothing technique for mathematical programs with equilibrium constraints is applied. Thus, a multiobjective single-level nonlinear programming problem is formalized. For solving this multiobjective single-level optimization problem, the scalarization approaches based on weighted sum approach and Tchebycheff approach are used respectively, and a constrained multiobjective differential evolution algorithm based on decomposition is presented. Some illustrative numerical examples including linear and nonlinear versions of MOBLPPs with multiple objectives at the upper level are tested to show the effectiveness of the proposed approach. Besides, NSGA-II is utilized to solve this multiobjective single-level optimization model. The comparative results among weighted sum approach, Tchebycheff approach, and NSGA-II are provided.

1. Introduction

Bilevel programming problem (BLPP) is a complicated mathematical model with a hierarchical structure involving two decision makers in the decision process [1]. BLPPs have a wide domain of applications, particularly in urban traffic and transportation, resource assignment, supply chain planning, structural optimization, engineering design, game playing strategies, and others. For example, Chiou [2] established a bilevel model with link capacity expansion for a normative road network design with uncertain travel demand in order to simultaneously reduce travel delay to road users and mitigate the vulnerability of the road network. Dempe et al. [3] developed a linear bilevel model for a natural gas cash-out problem between a natural gas shipping company and a pipeline operator. A penalty function method was proposed to solve the model. Hesamzadeh and Yazdani [4] proposed a mixed-integer linear bilevel model with multi-follower for transmission planning in an imperfect competition environment of the electricity supply industry, and the model was solved using Kuhn-Tucker optimality conditions and a binary mapping approach. Gao et al. [5] proposed two nonlinear bilevel pricing models for pricing problems between the vendor and the buyer in a two-echelon supply chain. A PSO-based algorithm was developed to solve these bilevel pricing models. Lots of instances in application have been summarized in [15–21].

In view of the fact that the applications of BLPPs are more and more extensive and diverse, efficient solution strategies are of critical importance for solving these BLPP models. Till now, many studies on solution strategies including classical methods and heuristic algorithms have been done for all types of BLPPs. Especially, a variety of heuristic algorithms has been employed to solve BLPPs successfully [6–14], which have numerous advantages, such as simplicity, efficiency, flexibility and robustness. Compared with classical methods, heuristic algorithms are suitable for either large-scale BLPPs or BLPPs with weak features. The reviews, monographs, and surveys on the models, algorithms and applications of BLPPs may refer to [1,15–21].

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Table 1
The average CPU time (in seconds) used by MOEA/D-DE with weighted sum approach and MOEA/D-DE with Tchebycheff approach.

Instance	Weighted sum approach	Tchebycheff approach
1	288.8705	269.1355
2	290.4008	305.5410
3	1153.4208	1169.1095
4	1514.5671	1652.7553
5	316.4801	381.0618
6	344.4778	256.3112
7	487.5152	579.9305
8	235.3970	219.2462
9	608.9422	550.5114
10	674.3858	691.0868
11	1060.9020	1102.1321

Table 2
The C-metric values between MOEA/D with weighted sum approach (A) and Tchebycheff approach (B). Mean denotes the mean value of C-metric values, and SD means the standard deviation of C-metric values in ten independent runs.

Instance	$C(A, B)$		$C(B, A)$	
	Mean	SD	Mean	SD
1	0.0099	0.0072	0.0251	0.0449
2	0	0	0	0
3	0.0106	0.1979	0.0053	0.0107
4	0.0078	0.0068	0.0086	0.0055
5	0.0040	0.0034	0.0043	0.0035
6	0	0	0.0205	0.0649
7	0.0120	0.0400	0.0205	0.0679
8	0.0517	0.0182	0.0788	0.0148
9	0.0503	0.0214	0.0768	0.0154
10	0.0009	0.0008	0	0
11	0.0022	0.0034	0.0702	0.0433

Multiobjective bilevel programming problem (MOBLPP) involving multiple objectives either at a certain level or at both levels has great significance in application, for example transportation system planning and management [22], network flow problem in a large-scale construction project [23]. However, in contrast with the vast literature on the BLPPs, little work has been conducted on MOBLPPs, either in algorithm or in application [25,26]. MOBLPPs can be classified into three categories: 1) MOBLPP with multiple objectives at the upper level [24–27], 2) MOBLPP with multiple objectives at the lower level [28,29], and 3) MOBLPP with multiple objectives at both levels [30–36]. Such multiobjective bilevel models are difficult to solve due to their intrinsic nonconvexity and many objectives even in one level. This paper centers on the solution methodology for MOBLPP in first category.

With respect to some recent studies on MOBLPP with multiple objectives at the upper level, most of the work focused on linear MOBLPPs. Ye [24] derived necessary optimality conditions by considering a combined problem, with both the value function and the Karush-Kuhn-Tucker (KKT) conditions of the lower-level problem involved in the constraints. Alves [25] proposed a multi-objective particle swarm optimization (MOPSO) algorithm to solve linear multiobjective bilevel programming problems with multiple objectives at the upper level. In MOPSO algorithm, each particle of the swarm is composed by two different parts, i.e. the upper level variable updated according to the principles of PSO, and the lower level variable given afterwards through the resolution of the lower-level optimization problem for the fixed upper level variable. Alves, Dempe, and Júdice [26] analyzed the linear bilevel programming problem with bi-objective on the upper level and a single objective at the lower level. However the problem considered in the paper has no lower level variables in the upper level constraints. The original problem was reformulated as a multiobjective mixed 0-1 linear programming problem. An existing interactive reference point procedure for multiobjective mixed-integer linear programming was employed to compute Pareto optimal solutions to the original problem. Calvete and Galé [27] considered general bilevel problems with many objectives at the upper level, when all objective functions are linear and constraints at both levels define polyhedra. This problem can be reformulated as a multiobjective problem with linear objective functions over a feasible region which is implicitly defined by a linear optimization problem and, in general, is non-convex. The weighted sum scalarization methods and scalarization methods were used to obtain efficient solutions. In above-mentioned literature, the solution algorithms were proposed only for linear version of MOBLPP with multiple objectives at the upper level. The aim of this paper is the development of solution methodology for both linear and nonlinear versions of MOBLPPs with multiple objectives at the upper level.

Regarding solution methodology of MOBLPP with multiple objectives at the upper level, there exist two ways to be chosen. One way is to transform the two-level structure to a single-level formation by adopting the optimality conditions or other techniques to the lower level optimization problem, and then utilize multiobjective evolutionary algorithms (MOEAs) or scalarization approaches to solve the single-level transformation model. For example, the way was used in [26]. The other way is to keep the two-level structure of original problem, and then apply MOEAs or scalarization approaches to the upper level optimization, while use the classical optimization techniques or heuristic algorithms to solve interactively the lower level optimization for each given upper level variable. For example, the way was used in [25]. The first way has a good efficiency, yet the lower level optimization must satisfy a certain optimality. In contrast, the second way is time-consuming, but the lower level optimization may have weak property. This paper aims at the first way in designing the algorithm for MOBLPP with multiple objectives at the upper level.

Based on above consideration, a solution approach for MOBLPP with multiple objectives at the upper level is presented. When the KKT optimality conditions are satisfied for the lower level optimization, the original multiobjective bilevel formulation can be converted into a multiobjective single-level nonlinear optimization problem with the complementarity constraints. Subsequently, an existing smoothing technique is applied to deal with the complementarity constraints. Thus, a constrained multiobjective single-level nonlinear optimization problem is formalized. For solving the reformulation of the original problem as a constrained multiobjective single-level programming problem, the scalarization approaches based on weighted sum approach and Tchebycheff approach are used respectively, and a constrained multiobjective differential evolution algorithm based on decomposition is presented, which is a modification to MOEA/D [39]. In addition, NSGA-II [41] is also utilized to solve reformulation of the original problem. By comparison of different MOEAs, we try to find which MOEA is more suitable for the reformulation of MOBLPP.

The main contributions of this work can be summarized as follows.

(1) The transformation model of MOBLPP is constructed to reduce its computational complexity. A multiobjective single level optimization is formed by using the KKT optimality conditions in the lower level programming, and then adopting the smoothing technique for the complementarity constraints.
(2) A constrained multiobjective differential evolution algorithm based on decomposition is developed for solving the transformation model of MOBLPP. For obtaining a uniform distribution of solutions in objective space, an adaptive weight