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A unified motion planning method for parking an autonomous vehicle in the presence of irregularly placed obstacles

Bai Li [a], Zhijiang Shao [a,b,*]

[a] Department of Control Science & Engineering, Zhejiang University, Hangzhou 310027, PR China
[b] State Key Laboratory of Industrial Control Technology, Hangzhou, PR China

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ABSTRACT

This paper proposes a motion planner for autonomous parking. Compared to the prevailing and emerging studies that handle specific or regular parking scenarios only, our method describes various kinds of parking cases in a unified way regardless they are regular parking scenarios (e.g., parallel, perpendicular or echelon parking cases) or not. First, we formulate a time-optimal dynamic optimization problem with vehicle kinematics, collision-avoidance conditions and mechanical constraints strictly described. Thereafter, an interior-point simultaneous approach is introduced to solve that formulated dynamic optimization problem. Simulation results validate that our proposed motion planning method can tackle general parking scenarios. The tested parking scenarios in this paper can be regarded as benchmark cases to evaluate the efficiency of methods that may emerge in the future. Our established dynamic optimization problem is an open and unified framework, where other complicated user-specific constraints/optimization criteria can be handled without additional difficulty, provided that they are expressed through inequalities/polynomial explicitly. This proposed motion planner may be suitable for the next-generation intelligent parking-garage system.

1. Introduction

Autonomous vehicles (sometimes called self-driving cars or driverless cars) refer to robotic vehicles that travel between destinations without human operators [1]. Such vehicles are expected to bring a variety of benefits, e.g., improving road network capacity and freeing up driver-occupants' time [2]. One industry analyst firm, Navigant Research, predicted that 75% of the vehicles sold in 2035 will have some sort of autonomous capability [3]. Although fully autonomous vehicles will not travel on the streets in the near future (because of the lack of legislation and mature technologies), yet the commercial availability of local vehicular automation systems (i.e., driver assistance systems and semi-autonomous systems) is increasing [4].

Autonomous parking is a critical application of driver assistance technologies. Relevant products have been designed by car manufacturers such as Audi, BMW, Ford, Land Rover, Mercedes-Benz, Nissan, and Toyota [5]. Nevertheless, these products are challenged in terms of thoroughly easing parking burdens. For instance, recognizing the environment during heavy rainstorms, inducing smart maneuvers to park in a narrow spot or grasping user preferences remains to be difficult issues [6,7]. In this sense, autonomous parking technologies deserve further investigation.

A successful autonomous parking process involves three sequential procedures: circumstance recognition, open-loop motion planning and closed-loop control execution [8]. Among these three procedures, motion planning alone is responsible for decision-making. In other words, the motion planning procedure largely determines how intelligent the entire parking system will be. Therefore, it is necessary to develop a reliable method in the motion planning phase.

Motion planning research studies in autonomous parking originated with [9], which systematically formulated a generalized autonomous parking problem for the first time. Ref. [10] categorized the prevailing motion planning algorithms into two types that are respectively applied in environments with complete or incomplete knowledge. Although many studies focus on motion planning in environments with incomplete knowledge [11], we believe that methods based on complete knowledge of the environment are not fully mature (the reasons will be presented later). This current study is based on an assumption that knowledge of the environment should be completely available before the motion planning procedure is implemented.

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* Corresponding author at: Department of Control Science & Engineering, Zhejiang University, Hangzhou 310027, PR China.

E-mail addresses: libai@zju.edu.cn (B. Li), szj@zju.edu.cn (Z. Shao).

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The prevailing motion planning methods on the basis of complete environmental knowledge can be broadly classified into three categories: geometric-based methods, heuristic-based methods, and methods based on control theories. Geometric-based approaches commonly compute reference paths first and then generate trajectories following the obtained paths (e.g., [12–16]). Here, a path refers to a geometric curve $y = f(x)$ in the xy coordinate frame, whereas a trajectory attaches the time course along a path, i.e., the determination of $x = x(t)$ [17]. Heuristic-based methods usually seek solutions from artificial intelligence techniques, e.g., fuzzy logics [18,19], search-based methods [20,21], random sampling methods [22] and machine learning methods [23]. Commonly the heuristic methods determine merely paths rather than trajectories, thus additional efforts must be exerted to convert the computed paths into trajectories. References regarding control theories are relatively scarce [24–26]. Such analytical methods usually deal with specific cases only, lacking generalization abilities [21]. Most of the previous publications mentioned above have validated their concerned methods effective through simulations, and some of those methods have even been executed on real robots in the field (e.g., [18,19]). In spite of their success, three issues stil deserve consideration. First, many existing methods do not solve the motion control problem directly. Typically, those heuristic-based path planning methods suffer from this limitation because kinematic descriptions of the vehicle are either missing or incomplete (e.g., [15,16,19–21]). In fact, quite few works have formulated complete kinematics (e.g., [27]). Second, it is better to generate optimal/optimized motions (based on some predefined criteria) rather than generate merely feasible motions. Third, we notice that a parking spot has been assumed as a slot region (see Fig. 1(a)) in most of the previous publications. The requirement that a car should not collide with the shaded regions in Fig. 1(a) is impractical. In fact, we only need the car terminally stay inside a rectangular parking spot. That is to say, the car can temporarily "invade" a neighboring spot region during its parking maneuvers provided that no collision happens. On the other hand, even when one is reluctant to invade temporarily into others' parking regions, he may find his target parking spot partly occupied by a parked car. Such parking scenarios (see Fig. 1(b)) are irregular but indeed ordinary in our daily life. Research studies that considered general parking scenarios are scarce. Apart from Paromtchik & Laugier's three publications in the early years (i.e., [28–30]), no other reverent studies can be found, to the best of our knowledge. As a brief summary, no study has solved or can solve the aforementioned three issues altogether.

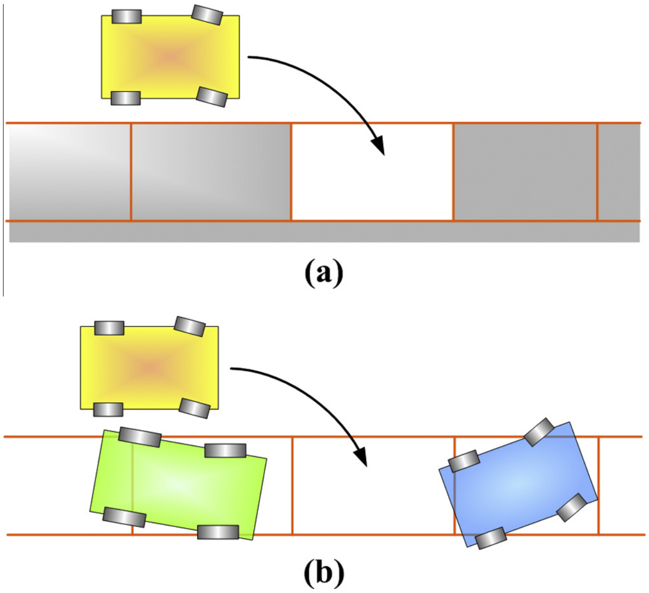

Fig. 1. Schematic of regular and irregular parking scenarios: (a) collision-free requirements in previous studies where a car should not hit the shaded regions during its parking maneuvers and (b) collision-free requirements considered in this current study, where a vehicle only needs to avoid colliding with neighboring cars during its parking maneuvers.

This work aims to address the original motion planning problem directly. To this end, differential equations are formulated to describe the vehicle kinematics and geometric analyses are conducted to strictly constrain the vehicle from hitting surrounding cars regardless they are regularly parked or not. We pursue for the time-optimal motions, thus formulating an optimal control problem (also can be regarded as a dynamic optimization problem) which is identical to the original parking motion planning scheme. A simultaneous approach based on interior point method (IPM) is applied to solve the formulated dynamic optimization problem.

The rest of this paper is organized as follows. In Section 2, the kinematics of an autonomous vehicle and the collision-free requirements are presented so as to formulate a dynamic optimization problem. In Section 3, the IPM-based simultaneous approach is introduced. In Section 4, simulations on several parking scenarios are presented, followed by Section 5, where detailed analyses on the simulation results are provided. Finally in Section 6, our conclusions are drawn.

2. Dynamic optimization problem formulation

This section formulates a dynamic optimization problem on the basis of the original parking motion planning mission. Detailedly, the vehicle kinematics, mechanical constraints and collision-free constraints will be introduced respectively. At the end of this section, we will show the overall formulation.

2.1. Kinematics of a car-like vehicle

The kinematics of a concerned front-steering autonomous vehicle can be expressed by

$$\begin{cases} \frac{\mathrm{d}x(t)}{\mathrm{d}t} = v(t) \cdot \cos\theta(t) \\ \frac{\mathrm{d}y(t)}{\mathrm{d}t} = v(t) \cdot \sin\theta(t) \\ \frac{\mathrm{d}v(t)}{\mathrm{d}t} = a(t) \\ \frac{\mathrm{d}\theta(t)}{\mathrm{d}t} = \frac{v(t) \cdot \sin\phi(t)}{l} \\ \frac{\mathrm{d}\phi(t)}{\mathrm{d}t} = \omega(t) \end{cases}, \tag{1}$$

where $t \in [0, t_f]$ refers to time, t_f indicates the terminal moment of the entire dynamic process, (x, y) refers to the mid-point of the front

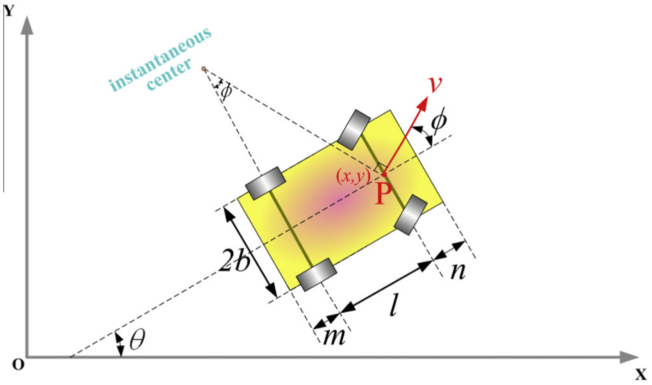

Fig. 2. Parametric notations related to vehicle size and kinematics.