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Period-adding bifurcation and chaos in a hybrid Hindmarsh-Rose model

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Abstract

Recently, the hybrid neuron models which combine the basic neuron models with impulsive effect(the state reset process) had been proposed, however, the preset value and the reset value of membrane potential were both fixed constants in the known models. In this paper, the Hindmarsh-Rose neuron model with nonlinear reset process is presented where the preset value and the reset value of membrane potential are variable constants. We conduct a qualitative analysis in the vicinity of the equilibrium point or the limit cycle of the proposed system by using the theories of impulsive semi-dynamical systems. Firstly, the more detailed impulsive set and phase set are given, then using the fixed point of Poincaré map, the existences of order-1 and order-k ($k >= 2$) period solutions are investigated subsequently. Furthermore, numerical investigations including period-adding bifurcation, multiple attractors coexistence, switch-like behavior are presented to further describe the bifurcation and chaos phenomena. Finally, the obtained results and possible applications of the proposed model are elaborated.

Keywords: Hindmarsh-Rose, Impulsive effect, Poincaré map, Period-adding bifurcation, Chaos

1. Introduction

The scientific researchers have poured a great endeavor in finding neuron model to accord with the nature of realistic nerve cell, the obtained model should not only be close to realistic nerve cell, but also be facilitated in computation. Although the Hodgkin-Huxley model(HH) had been proposed as early as 1952(Hodgkin and Huxley, 1952) to describe how action potentials in neurons are initiated and propagated, but the Hodgkin-Huxley model can be thought of a differential equation with four state variables that change with respect to time. The system is difficult to study and cannot be solved analytically. Subsequently, Hindmarsh and Rose(Hindmarsh and Rose, 1984) in 1984 have reduced HH neuron model to the nonlinear differential equations:

$$\begin{cases} \dot{x}(t) &= y(t) - ax(t)^3 + bx(t)^2, \\ \dot{y}(t) &= c - dx(t)^2 - y(t), \end{cases} \tag{1}$$

where x is the membrane potential, y represents the transport of sodium and potassium ions through fast ion channels, which is called the fast spiking variable. a, b, c, d are parameters. The Hindmarsh-Rose model has displayed most of the Hodgkin and Huxley neuronal model's characteristics, they also attempted to provide a simpler way to simulate synchronization of firing of two snail neurons after the formulation of this two-equation system, consequently, Hindmarsh-Rose's three-equation model was proposed to substitute for the two-dimensional model by adding a slow adaptation bursting variable, which is favorite in studying biological properties of spiking and bursting neurons.

Over the past decades, the increasing interest was devoted to the work of complex dynamics of neuron models and neural networks. In addition to fundamental properties of stability, passivity, convergence, bifurcation phenomena (Liao et al., 2002; Huang et al., 2013b; Wu and Zeng, 2014b,c, 2016a, 2017; Wu et al., 2017; Wu and Zeng, 2014a, 2016b; Dong et al., 2012; Dong and Liao, 2013), there were also many researchers considered other extension problems, such as synchronization problems(Huang et al., 2009; Li et al., 2014), classification problem(Mcloughlin et al., 2017; Qian et al., 2017), state estimation(Wang and Song, 2010; Huang et al., 2013a) optimization problems(Li et al., 2017, 2016), etc. In terms of Hindmarsh-Rose systems, their works mainly focused on theoretical and numerical methods of the bifurcation and chaos analysis(Li and He, 2014; Shilnikov and Kolomiets, 2008; Liu and Liu, 2012; Gonzlez-Miranda, 2011) and the synchronization issue of Hindmarsh-Rose model(Sanjaya et al., 2012; Checco et al., 2009; Hrg, 2013). Nevertheless, the Hindmarsh-Rose model was seldom considered for impulsive effect except the spiking and bursting phenomena generated by the system itself. The impulsive effect was usually called discontinuous reset process in neuron models(Izhikevich, 2003; Nobukawa et al., 2015; Zheng and Tonnelier, 2009; Nobukawa et al., 2016). In Izhikevich's article(Izhikevich, 2003), a two-dimensional system of ordinary differential equations with auxiliary after-spike reset was proposed, where the threshold value of membrane potential was set at 30mV(the peak of the spike), when the membrane potential reached at 30mV, the membrane potential was reset at a constant c, which had accounted for the transition from saddle-node on invariant circle to saddle-node separatrix loop. Inspired by the above relative works, we try to

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