Neural Networks 105 (2018) 154–165

Contents lists available at ScienceDirect

Neural Networks

journal homepage: www.elsevier.com/locate/neunet

# Finite-time synchronization of stochastic coupled neural networks subject to Markovian switching and input saturation

P. Selvaraj [a], R. Sakthivel [b,c,*], O.M. Kwon [a,**]

[a] School of Electrical Engineering, Chungbuk National University, 1 Chungdao-ro, Cheongju 28644, South Korea
[b] Department of Mathematics, Bharathiar University, Coimbatore 641046, India
[c] Department of Mathematics, Sungkyunkwan University, Suwon 440-746, South Korea

ARTICLE INFO

*Article history:*
Received 23 January 2018
Received in revised form 11 April 2018
Accepted 4 May 2018

*Keywords:*
Coupled stochastic neural networks
Finite-time synchronization
Saturation effect
Markovian jumping parameters
Stochastic coupling strength

ABSTRACT

This paper addresses the problem of finite-time synchronization of stochastic coupled neural networks (SCNNs) subject to Markovian switching, mixed time delay, and actuator saturation. In addition, coupling strengths of the SCNNs are characterized by mutually independent random variables. By utilizing a simple linear transformation, the problem of stochastic finite-time synchronization of SCNNs is converted into a mean-square finite-time stabilization problem of an error system. By choosing a suitable mode dependent switched Lyapunov–Krasovskii functional, a new set of sufficient conditions is derived to guarantee the finite-time stability of the error system. Subsequently, with the help of anti-windup control scheme, the actuator saturation risks could be mitigated. Moreover, the derived conditions help to optimize estimation of the domain of attraction by enlarging the contractively invariant set. Furthermore, simulations are conducted to exhibit the efficiency of proposed control scheme.



## 1. Introduction

Network systems have recently received considerable attention due to their increasing demand on many real-time systems (Hunt, Irwin, & Warwick, 1995; Peng, Yue, & Han, 2015). More specifically, the study of neural networks has gained significant attention because it is not only a question of theoretical interest to mathematicians but also plays an indispensable role in broad range of practical applications in various fields, such as neural circuit in brain, food web in ecosystem and internet in communication networks (Ahn, Shi, & Wu, 2015; Park, Kwon, & Seuret, 2017; Zineddin, Wang, & Liu, 2011). Though several studies on neural networks, the actual impact of coupling behaviour in such systems is just beginning, much work to be done. In particular, coupled neural networks (CNNs) may exhibit more complicated and unpredictable behaviours, but their analysis helps to imitate system behaviours better than the traditional neural networks (Manivannan, Samidurai, Cao, Alsaedi, & Alsaadi, 2018; Sakthivel, Anbuvithya, Mathiyalagan, Ma, & Prakash, 2016; Wang, Teng, & Jiang, 2012; Zheng & Cao, 2014). On the other hand, as a typical collective behaviour of networks, synchronization has received increasing attention (Cai, Huang, & Zhang, 2017; Fang & Park, 2013; Huang, Li, Huang, & He, 2014; Lee, Park, Kwon, & Sakthivel, 2016; Xu, Wang, & Wei, 2017; Zhang & Gao, 2017). To date, most of the existing literature results related to the synchronization analysis of CNNs, the coupling strength has been assumed to be deterministic. This assumption is, however, heavily limited as almost all real-world network applications because of the complicated environment nature. Indeed, the coupling strength of networks is stochastic. Moreover, the dynamics of neural networks consist of a vast myriad of interacting components, whose internal details are too probabilistic. Nonetheless, the study of synchronization of stochastic coupled neural networks (SCNNs) is still in its infancy (Bao, Park, & Cao, 2016; Samidurai & Manivannan, 2016).

Meanwhile, the realistic modelling of many neural networks inevitably needs to take into account of random abrupt state switching caused by the sudden environment changes or the structural and parametric changes of systems. But, the dynamics of neural networks with random switching cannot be appropriately described by the commonly used linear time-invariant state space representation. However, the switching signal between different neural network modes can be represented by using the Markov process. In Markovian type pattern, the future action depends only on the current action not all past actions, so it requires only short memories, for instance see Li, Shi, and Wu (2017) and Wang, Zhang, and Yan (2015). In addition, Markov process is highly suitable for many real-time networks, such as communication networks, electric networks, and aerospace networks (Li, Shi, Wu, Basin, &

* Corresponding author at: Department of Mathematics, Bharathiar University, Coimbatore 641046, India.
** Corresponding author.
*E-mail addresses:* krsakthivel@buc.edu.in (R. Sakthivel), madwind@chungbuk.ac.kr (O.M. Kwon).

https://doi.org/10.1016/j.neunet.2018.05.004
0893-6080/

Lim, 2015; Liu, Wang, Liang, & Liu, 2013; Yang, Cao, & Lu, 2013; Zhu & Cao, 2011, 2012). In Park, Kwon, Park, and Lee (2012), a sufficient condition has been derived in the framework of linear matrix inequalities (LMIs) for ensuring the stability of a class of fuzzy Markovian jumping Hopfield neural networks described by neutral-type differential equations.

Actuator saturation is one of the most important non-smooth input nonlinearities in the control system design and analysis. For some highly complex network systems, it is difficult to deliver arbitrarily large signals through real actuators. So, neglecting the actuator saturation phenomenon can consequently be the source of unwanted or even catastrophic actions in the closed-loop system. The study of control analysis with actuator saturation initiated for the past few decades, but recently, research in this direction has seen a revival in neural network systems (Wang, Qiu, & Gao, 2017; Wu, Shi, Su, & Chu, 2014; Wu, Su, & Wu, 2015). One widely adopted method for handling the actuator saturation is anti-windup control scheme. It was observed that when the control input signal reaches beyond the saturation limits the control input signal wind up, which leads difficulty to fulfil the control objective by using classical linear feedback controllers. Therefore, to tolerate that negative performance due to wind up effect is tolerated with the aid of anti-wind up control scheme. Under this control scheme, the control signal is represented the linear combination of actual and auxiliary linear feedback signals, which reveals the highly better performance compared with the conventional control performance against wind up effect (Li, Wang, & Shi, 2016; Pan, Sun, Gao, & Jing, 2016; Selvaraj, Kaviarasan, Sakthivel, & Karimi, 2017; Wu et al., 2014; Yang, Zhang, & Sun, 2016).

On another research frontier, synchronization is one of the fundamental tasks in network systems because of its immense applications in various fields, such as communication networks, biological networks, chemical reactions and heartbeat regulation. To date, various kinds of synchronization problems, such as asymptotic synchronization, exponential synchronization, pinning distributed synchronization and finite-time synchronization have been investigated in the literature (Chen, Shi, & Lim, 2017; Tang, Gao, Lu, & Kurths, 2014; Wu, Shi, Su, & Chu, 2013). Though there have been many results concerning the asymptotic synchronization and exponential synchronization of network systems, in which the synchronization can only be realized when time tends to infinity, many practical applications need to achieve the synchronization as quickly as possible and to guarantee the fast response. To shorten the convergence time of error states still further, an effective technique is finite-time synchronization or finite-time boundedness (Liu, Ho, Cao, & Xu, 2017; Liu, Ho, Song, & Cao, 2017; Mathiyalagan, Park, & Sakthivel, 2016; Wang & Zhu, 2015; Wu, Cao, Li, Alsaedi, & Alsaadi, 2017; Yang & Huang, 2017). In addition, finite-time boundedness condition possesses some additional nice features with finite-time synchronization, such as better robustness and disturbance rejection properties. Based on these facts and demonstrations, finite-time synchronization is more powerful in neural networks.

Inspired by the above discussion, in this paper, by utilizing Lyapunov stability theory, we examine the finite-time synchronization problem of SCNNs with Markovian jump parameters, mixed time-varying delays and actuator saturation effect. The main contributions of this paper are highlighted as follows:

1. By taking the effects of actuator saturation and stochastic coupling strength into account, a new anti-windup state feedback controller is proposed for SCNNs. Also, the sufficient conditions are established to verify the finite-time synchronization of SCNNs. Therefore, the proposed controller is more suitable to apply for many practical network systems.
2. The proposed synchronization criteria are dependent not only on all the mode delays but also on the generator of the Markovian chain, stochastic noise, mathematical expectation, and variance of the random coupling strength variables.
3. Based on the proposed criterion, the state-feedback controller is designed and an optimal estimation of attraction regions is obtained to the feasible solutions of a set of LMI constraints.

Finally, a numerical example is provided to demonstrate the effectiveness of the proposed theoretical results.

**Notation:** Throughout this paper, the used notations are standard. $\mathbb{R}^n$ denotes the $n$-dimensional Euclidean vector space, and $\mathbb{R}^{m\times m}$ is the set of all $m \times m$ real matrices. If $\mathcal{P}$ is a symmetric matrix, $\lambda_{max}(\mathcal{P})$ and $\lambda_{min}(\mathcal{P})$ denote the maximum and minimum eigenvalues of $\mathcal{P}$, respectively. $\mathcal{I}$ represents the identity matrix with appropriate dimensions. The Kronecker product of matrices $\mathcal{A} \in \mathbb{R}^{l\times n}$ and $\mathcal{B} \in \mathbb{R}^{p\times q}$ is a matrix in $\mathbb{R}^{lp\times nq}$ and denoted as $(\mathcal{A} \otimes \mathcal{B})$. The asterisk '$*$' represents a term that is induced by symmetry. $\mathbb{E}\{\cdot\}$ denotes the mathematical expectation and $q_l$ denotes $\left[0_{n\times(l-1)n}, I_n, 0_{n\times(26-l)n}\right]$. $Pr\{\cdot\}$ represent the probability function. Denote the sets $\Omega(P,\delta) = \{x \in \mathcal{R}^n : x^T(t)Px(t) \le \delta\}$ and $\mathcal{L}(F) := \{x \in \mathbb{R}^n : |f_l x| \le 1,\ l = 1, 2, \ldots, n\}$ as an ellipsoid and a polyhedral, respectively, where $P$ is a positive definite matrix, and $f_l$ is the $l$th row of the matrix $F \in \mathbb{R}^{n\times n}$.

## 2. Preliminaries and problem formulation

### 2.1. Mathematical preliminaries

To introduce some basic strategy of reconstructing networks, we first consider the stochastic system in the following form:

$$\mathrm{d}x(t) = \alpha(x(t), r(t))\mathrm{d}t + \varsigma(x(t), r(t))\mathrm{d}w(t), \tag{1}$$

where $\alpha(\cdot)$ and $\varsigma(\cdot)$ are vector valued nonlinear functions, $x(t)$ is the state vector, $\{w(t), t \ge 0\}$ stands for an $n$-dimensional standard Brownian motion defined on a complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\ge 0}, \mathcal{P})$, where $\Omega$ is a sample space, $\mathcal{F}$ is a field with the natural filtration $\{\mathcal{F}_t\}_{t\ge 0}$, and $\mathcal{P}$ is a probability measure on $\mathcal{F}$. Moreover, the Brownian motion $\{w(t), t \ge 0\}$ satisfies $\mathbb{E}\{\mathrm{d}w(t)\} = 0$ and $\mathbb{E}\{\mathrm{d}w(t)^2\} = \mathrm{d}t$. Further, the random jumping process $\{r(t), t \ge 0\}$ is a right-continuous Markovian process taking values in a finite set $\mathbb{S} = \{1, 2, \ldots, M\}$ in the mode transition probabilities space $(\Omega', \mathcal{G}, \mathbb{P})$, where $\Omega'$ is the sample space, $\mathcal{G}$ is the algebra of events and $\mathbb{P}$ is the probability measure defined on $\mathcal{G}$. The mode transition probabilities are described as follows:

$$Pr\{r(t+dt) = q | r(t) = p\} = \begin{cases} \pi_{pq} d(t) + O(dt), & \text{if } p \neq q, \\ 1 + \pi_{pq} dt + O(dt), & \text{if } p = q, \end{cases}$$

where $dt > 0$ and $\lim_{dt\to 0}\frac{O(dt)}{d(t)} = 0$, $\pi_{pq} \ge 0$ is the transition rate from mode $p$ at time $t$ to mode $q$ at time $t + dt$ if $p \neq q$ and $\pi_{pp} = -\sum_{q=1,p\neq q}^{M}\pi_{pq}$, $\forall p \in \mathbb{S}$. Moreover, we assume that the Brownian motion $\{w(t), t \ge 0\}$ is independent from the Markov chain $\{r(t), t \ge 0\}$.

### 2.2. Problem formulation

Consider a class of stochastic Markovian jump coupled neural networks with mixed mode-dependent time-varying delays and