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Global exponential stability of octonion-valued neural networks with leakage delay and mixed delays

Călin-Adrian Popa

Department of Computer and Software Engineering, Polytechnic University Timişoara, Blvd. V. Pârvan, No. 2, 300223 Timişoara, Romania

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ABSTRACT

This paper discusses octonion-valued neural networks (OVNNs) with leakage delay, time-varying delays, and distributed delays, for which the states, weights, and activation functions belong to the normed division algebra of octonions. The octonion algebra is a nonassociative and noncommutative generalization of the complex and quaternion algebras, but does not belong to the category of Clifford algebras, which are associative. In order to avoid the nonassociativity of the octonion algebra and also the noncommutativity of the quaternion algebra, the Cayley–Dickson construction is used to decompose the OVNNs into 4 complex-valued systems. By using appropriate Lyapunov–Krasovskii functionals, with double and triple integral terms, the free weighting matrix method, and simple and double integral Jensen inequalities, delay-dependent criteria are established for the exponential stability of the considered OVNNs. The criteria are given in terms of complex-valued linear matrix inequalities, for two types of Lipschitz conditions which are assumed to be satisfied by the octonion-valued activation functions. Finally, two numerical examples illustrate the feasibility, effectiveness, and correctness of the theoretical results.

1. Introduction

Neural networks with values in multidimensional domains have attracted the attention of researchers over the last few years. First introduced by Widrow, McCool, and Ball (1975), complex-valued neural networks (CVNNs) have found numerous applications, which include antenna design, radar imaging, estimation of direction of arrival and beamforming, image processing, communications signal processing, and many others (Hirose, 2012, 2013). Quaternion-valued neural networks (QVNNs) were introduced by Arena, Fortuna, Occhipinti, and Xibilia (1994), and have applications in chaotic time-series prediction (Arena, Fortuna, Muscato, & Xibilia, 1998), color image compression (Isokawa, Kusakabe, Matsui, & Peper, 2003), color night vision (Kusamichi, Isokawa, Matsui, Ogawa, & Maeda, 2004), polarized signal classification (Buchholz & Le Bihan, 2008), and 3D wind forecasting (Jahanchahi, Took, & Mandic, 2010; Took, Mandic, & Aihara, 2010). Clifford-valued neural networks (ClVNNs), proposed by Pearson and Bisset (1992, 1994), and later discussed by Buchholz and Sommer (2008) and Kuroe, Tanigawa, and Iima (2011), have potential applications in high-dimensional data processing. They represent a generalization of the complex- and quaternion-valued neural networks, because complex and quaternion algebras are special cases of the 2^n-dimensional Clifford algebras, where $n \geq 1$.

A different generalization of the complex and quaternion algebras is the octonion algebra. It is an 8-dimensional normed division algebra, which means that a norm and a multiplicative inverse can be defined on it. In fact, it is the only normed division algebra that can be defined over the field of real numbers, besides the complex and quaternion algebras. The octonion algebra is not a special kind of Clifford algebra, because the Clifford algebras are all associative, whereas the octonion algebra is not.

Octonions have applications in physics and geometry (Dray & Manogue, 2015; Okubo, 1995), and have also been successfully applied in the signal processing domain in the recent years (Snopek, 2015). The signal processing applications include salient object detection (Gao & Lam, 2014a, b), hyperspectral fluorescence data fusion (Bauer & Leon, 2016), L1-norm minimization for octonion signals (Wang, Xiang, & Zhang, 2016), and the octonion Fourier transform (Błaszczyk & Snopek, 2017). In physics, octonions were used to reformulate electrodynamics and chromodynamics equations (Chanyal, 2013; Chanyal, Bisht, Li, & Negi, 2012; Chanyal, Bisht, & Negi, 2010, 2011), the Maxwell equations (Demir & Tanişli, 2016), the gravitational field equations (Demir, 2012), and the Dirac equation (Koplinger, 2006).

Taking all the above facts into consideration, octonions may have potential applications in the neural network domain, also. Thus, feedforward octonion-valued neural networks (OVNNs) were first proposed by Popa (2016). They may be applied in the signal processing domain, where certain signals can be better

E-mail address: calin.popa@cs.upt.ro.

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represented in the octonion domain. High-dimensional data processing could also benefit from the use of octonion networks. In the same way as complex-valued networks were better than real-valued ones for some applications, and quaternion-valued networks were better than both real- and complex-valued networks in others, octonion-valued networks may outperform all of the above in yet other problems. They could represent an alternative for 8-dimensional Clifford-valued neural networks, because of the property of being a normed division algebra that the octonion algebra has. Thus, the multidimensional algebras can constitute a more general framework for neural networks, which could benefit not only from increasing the number of hidden layers and making the architecture ever more complicated, but also from increasing the dimensionality of the data that is being handled by the network. The multidimensional neural networks field is rather new, and it is expected that the future will bring even more applications for this type of networks.

The practical applications of neural networks heavily rely on their dynamical properties. To solve problems of optimization, neural control, and signal processing, neural networks have to be designed in such way that they exhibit only one globally stable steady state. Thus, sufficient conditions can be determined, which depend on the system parameters, that guarantee the existence of a unique globally stable steady state for a certain neural network.

The stability properties of CVNNs were intensely studied in recent years. The global asymptotic and exponential stability for CVNNs with time delays were discussed by Hu and Wang (2012), both by decomposing the complex numbers into their two real components and directly in the complex domain. Sufficient conditions for the stability of CVNNs with both leakage time delay and discrete time delay on time scales were established by Chen and Song (2013), by decomposition. Several conditions in terms of real-valued linear matrix inequalities (LMIs) for the boundedness and complete stability of CVNNs were developed by Zhou and Song (2013). CVNN models with mixed delays were considered by Xu, Zhang, and Shi (2014), for which exponential stability criteria were given, also using the decomposition method. The complete stability for CVNNs with time delays and impulses was the concern of Rakkiyappan, Velmurugan, and Li (2014), who used both decomposition and direct methods to undertake the analysis. Zhang, Lin, and Chen (2014) gave a sufficient criterion for the global asymptotic stability of delayed CVNNs in terms of real-valued LMIs. New criteria for the existence, uniqueness, and global asymptotic stability of the equilibrium point of CVNNs with time delays were established by Fang and Sun (2014), in terms of complex-valued LMIs.

Chen, Song, Liu, and Zhao (2014a, b) studied the global μ-stability of CVNNs with unbounded time-varying delays and with leakage delay, mixed delays, and impulses, respectively, both in the real and complex domains. The problem of global μ-stability was further analyzed by Velmurugan, Rakkiyappan, and Cao (2015), and by Gong, Liang, and Cao (2015). Sufficient conditions for the exponential stability of a class of CVNNs with time-varying delays were also developed by Pan, Liu, and Xie (2015), by using a delay differential inequality. CVNNs with time-varying delays and impulsive effects were investigated by Song, Yan, Zhao, and Liu (2016a), who used the direct method to give sufficient criteria in terms of complex-valued LMIs for their exponential stability. A delay-dependent condition expressed in terms of complex-valued LMIs, which assures the global exponential stability for CVNNs with both leakage delay and time-varying delays on time scales was established by Song and Zhao (2016). Song, Yan, Zhao, and Liu (2016b) gave several sufficient conditions for the global exponential stability of impulsive CVNNs with both asynchronous time-varying and continuously distributed delays, which was also the focus of Liu and Chen (2016). New asymptotic stability criteria for delayed CVNNs were given by Zhang, Liu, Chen, Guo, and Zhou (2017), both in terms of real-valued and complex-valued LMIs. The same type of stability was studied by Subramanian and Muthukumar (2017), for CVNNs with additive time-varying delays. Discrete-time neural networks (Yang, Wu, & Liu, 2015) were also discussed in the complex domain by Chen, Song, Zhao, and Liu (2016), Hu and Wang (2015), Mostafa, Teich, and Lindner (2014) and Song, Zhao, and Liu (2015).

QVNNs have also attracted the attention of researchers very recently. The global μ-stability for QVNNs with unbounded time-varying delays was first studied by Liu, Zhang, Lu and Cao (2016), by splitting the quaternions in two complex numbers. QVNNs with time-varying delays were also discussed by Liu, Zhang, and Lu (2016), by decomposing the quaternions into their four real components, and using the Halanay inequality to deduce sufficient conditions for the exponential stability. Sufficient conditions for the robust stability of the equilibrium point for QVNNs were derived by Chen, Li, Song, Hu, and Tan (2017), directly in the quaternion domain, using a modulus inequality technique for the quaternions. The decomposition approach was used also by Zhang, Kou, Liu, and Cao (2017) to study the stability of QVNNs with asynchronous time delays. Shu, Song, Liu, Zhao, and Alsaadi (2017) used the same method as Liu, Zhang, Lu & Cao (2016) to extend the stability results for QVNNs with non-differentiable time-varying delays. Global μ-stability was also the concern of Liu, Zhang, Lou, Lu, and Cao (2017), who employed two methods to deduce sufficient stability conditions: one by decomposition, and the other one directly in the quaternion domain. Chen, Song, and Li (2017) used QVNNs to design associative memories in the quaternion domain. The authors show that the constructed QVNNs work efficiently on storing and retrieving blurred gray-scale and true color images. Multistability and multiperiodicity conditions for impulsive QVNNs with mixed delays were established by Popa and Kaslik (2018).

Lastly, ClVNNs are emerging as a field of study. The global stability of ClVNNs with time delays was discussed by Liu, Xu, Lu, and Liang (2015), by rewriting the Clifford-valued system of differential equations as a real-valued one. The same model, but without delays, was investigated by Zhu and Sun (2016), directly in the Clifford domain.

Taking all the above into account, the aim of this paper is to develop exponential stability criteria for OVNNs with leakage delay, time-varying delays, and distributed delays. To the best of our knowledge, this type of problem has not yet been discussed in the literature. Because of the finite switching speed of amplifiers, delays occur in real-life implementations of neural networks, and can cause unstable or oscillatory behavior. For this reason, we consider both bounded leakage delay, and bounded time-varying delays in the OVNN model. On the other hand, distribution propagation delays may appear as a consequence of a distribution of conduction velocities along the pathways of a neural network implementation, which compelled us to also add continuously distributed delays to our model.

Using the Cayley–Dickson construction, the octonion-valued system of equations that defines an OVNN is decomposed into 4 complex-valued differential systems, in order to avoid the nonassociativity of the octonion algebra and also the noncommutativity of the quaternion algebra. Thus, by defining appropriate Lyapunov–Krasovskii functionals with double and triple integral terms, using the simple and double integral complex-valued Jensen inequalities, and the free weighting matrix method, two stability criteria are given in terms of complex-valued LMIs, which can be easily solved using the effective YALMIP tool in MATLAB. The reason we chose the complex-valued decomposition is because, as can be seen from the above considerations, the complex-valued stability theory of neural networks is already an established field, and to avoid the cumbersome calculations implied by a decomposition of octonions into their 8 real components. Also, the two