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Generalized pinning synchronization of delayed Cohen–Grossberg neural networks with discontinuous activations[☆]

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ABSTRACT

In this article, generalized pinning synchronization problem is investigated for a class of Cohen–Grossberg neural networks with discontinuous neuron activations and mixed delays. By designing generalized pinning state-feedback and adaptive controllers, several criteria for global exponential synchronization and global asymptotical synchronization of the drive–response based system are obtained in view of non-smooth analysis theory with generalized Lyapunov functional method, in which first pinning the neurons with very small self-inhibition and small amplification functions is pointed out. Some numerical examples are given to illustrate the feasibility of the obtained results.

1. Introduction

As far as we know, the non-Lipschitz or discontinuous neuron activations widely exist in many practical neural networks. They are caused by some interesting engineering tasks, such as dry friction, impacting machines, power circuits, switching in electronic circuits (Chen, Wang, & Liu, 2000; Chong, Hui, & Zak, 1999; Tsang, Wang, & Yeung, 2000). Furthermore, McCulloch and Pitts pointed out that neural events and the relations among them can be treated by means of the two-valued logic of propositions due to the "all-or-none" character of nervous activity (McCulloch & Pitts, 1943). In general, a neural network with non-Lipschitz or discontinuous neuron activations is believed as an ideal model for neuron amplifiers with very high gains (Forti & Nistri, 2003). As a matter of fact, the sigmoidal neuron activations of the classical Hopfield network with high-gain amplifiers would approach a discontinuous hard-comparator function (Yang, Cao, & Ho, 2015). Hopfield and Tank also pointed out that the high-gain hypothesis is crucial to make negligible the contribution to neural network energy function of the term depending on neuron self-inhibitions, and to favor binary output formation (Hopfield & Tank, 1985). When dealing with neural networks possessing high-slope nonlinear activations, it is often advantageous to model them with a system of differential equations with discontinuous neuron activations, rather than with continuous ones whose slope is high but of finite value (Forti & Nistri, 2003; Yang et al., 2015). Thereupon, many important and interesting dynamical behaviors of neural networks with discontinuous neuron activations have been proposed and developed such as stability, periodicity and multi-periodicity (Forti, Grazzini, & Nistri, 2006; Forti, Nistri, & Papini, 2005; Liu, Liu, & Xie, 2012; Lu & Chen, 2005; Wang & Huang, 2014a, b, 2016; Wang & Luo, 2015; Wu, 2012).

Recently, the problem of synchronization control for neural networks has been one of hot research issues since the pioneering work of Pecora and Carroll (1990). It has received great attention due to its potential applications, such as secure communication, biological systems and information science (Kocarev, 2013;

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Lu, Liu, & Chen, 2016; Tang, Lu, & Chen, 2012; Tang, Lu, Lü, & Yu, 2012; Wu & Yang, 2016; Zhang, Wu, Lu, Feng, & Lü, 2014). When the discontinuous neuron activations are incorporated into neural networks, some interesting collective dynamical behaviors, such as quasi-synchronization, finite-time synchronization and global exponential synchronization can appear (Cai, Huang, & Zhang, 2015; Duan, Huang, & Fang, 2017; Liu, Cao, Yu, & Song, 2016; Liu, Chen, Cao, & Lu, 2011; Liu, Ho, Cao, & Xu, 2017; Wang, Huang, & Tang, 2015, 2018; Wu & Yang, 2016; Yang & Cao, 2013a, b). In Liu & Cao (2010), the complete synchronization was considered for a class of neural networks with discontinuous neuron activations via approximation approach. Using the state feedback control, Liu et al. (2012) studied the quasi-synchronization for a class of delayed neural networks with discontinuous neuron activations and parameter mismatches. By constructing suitable Lyapunov functional, Yang and Cao (2013a) investigated the exponential synchronization for a class of delayed neural networks with discontinuous neuron activations via state-feedback controllers and adaptive controllers, respectively. By designing discontinuous state-feedback controllers, Cai et al. (2015) studied two kinds of global exponential synchronization of the drive–response-based discontinuous neural networks. By designing continuous and discontinuous state feedback controllers, Wang et al. (2015) studied the complete synchronization for a class of neural networks with discontinuous neuron activations, in which the discontinuous neuron activations do not satisfy the Lipschitz-like condition. Wang et al. (2018) studied the global exponential synchronization for a class of neural networks with discontinuous neuron activations via delay-dependent, delay-independent and adaptive discontinuous controllers, respectively, in which the derivative of the time-varying delay can be considerably large, even the time-varying delay can be non-differentiable. So far, all those works on the issue of synchronization control have to add controllers to each neuron of the discontinuous neural networks. Generally, it is too costly and impractical to add controllers to all neuron in large-scale neural networks with discontinuous neuron activations.

Pinning control strategy, it means that one only needs to place the feedback injections on a small fraction of network nodes. As an effective and powerful method, pinning control strategy has been proposed in the study of synchronization for complex networks (Wang & Chen, 2002; Yu, Chen, Lü, & Kurths, 2013; Zhou, Lu, & Lü, 2008). So far, the problem of synchronization control for neural networks via pinning control strategy has received increasing attention (Song, Cao, & Liu, 2012; Wen, Yu, Hu, Cao, & Yu, 2015; Zhou, Lu, et al., 2008; Zhou, Wu, et al., 2008). However, many previous works on pinning control strategy focused mainly on the neural networks with continuous neuron activations, and their tools or methods are generally invalid for discontinuous cases due to the essential difference between continuous and discontinuous functions. Obviously, the research on synchronization of neural networks with discontinuous neuron activations via pinning control strategy is more difficult but more practical. In this case, we develop the generalized synchronization pinning scheme for neural networks with discontinuous neuron activations, and try to answer two basic problems: which neurons should be pinned first? and how large should the control strength be selected for more less control gains? This is the main contribution in this text.

On the other hand, Cohen–Grossberg neural network, an important recurrent neural networks model first described by Cohen and Grossberg (1981), has aroused a tremendous surge of investigation from various fields, such as parallel computation, signal and image processing, nonlinear optimization and pattern recognition. Recently, there are many works on delayed Cohen–Grossberg neural networks with discontinuous neuron activations (Lu & Chen, 2005; Wang & Huang, 2014a, b, 2016; Wang, Huang, & Cai, 2013; Wu, Zhang, & Li, 2015). Among these works, Wu et al. studied the exponential synchronization for a class of delayed Cohen–Grossberg neural networks with discontinuous activations under the assumption of discontinuous neuron activations being bounded and monotone non-decreasing. But when dealing with an unbounded dependent variable, one could choose an unbounded nonlinear activation function. As pointed out by Gonzalez (2000), a nonlinear activation function should be used for truly exploiting the potential of neural networks. Based on this point, using the unbounded and monotone discontinuous neuron activations is very interesting. For Cohen–Grossberg neural networks with unbounded discontinuous neuron activations, the main problem in synchronization investigations is how to handle the amplification function. Inspired by the idea of Chen and Rong (2004) and Lu and Chen (2003), we introduce the equivalent transformation method to deal with the amplification function, this is another contribution of this paper.

The rest of this paper is outlined as follows. Model description and some preliminaries concerning Filippov Solution are presented in Section 2. In Section 3, the equivalent transformation method is presented at first. After that, the generalized pinning control schemes with state-feedback controllers and adaptive controllers are designed. Based on functional differential inclusions theory and non-smooth analysis theory with generalized Lyapunov functions method, several criteria on global exponential synchronization and global asymptotical synchronization of the drive–response-based systems are derived. It is noteworthy that these controllers and non-smooth Lyapunov functions in this paper are essentially new and different from those in the earlier literatures (Cai et al., 2015; Duan et al., 2017; Liu et al., 2011; Wang et al., 2015, 2018; Wu & Yang, 2016; Wu et al., 2015; Yang & Cao, 2013a, b). In Section 4, numerical simulations further illustrate the effectiveness of our results. Section 5 presents a brief conclusion.

2. Neural networks and preliminaries

In this paper, we consider the following Cohen–Grossberg neural networks with discontinuous activations and mixed time delays:

$$\begin{aligned}\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} &= \alpha_i(x_i(t))\Big[-d_i(t)x_i(t) + \sum_{j=1}^{n} a_{ij}(t)f_j(x_j(t)) \\ &\quad + \sum_{j=1}^{n} b_{ij}(t)f_j(x_j(t-\tau(t))) \\ &\quad + \sum_{j=1}^{n} c_{ij}(t)\int_0^{+\infty} f_j(x_j(t-s))k(s)\mathrm{d}s + I_i(t)\Big], \\ &\quad i = 1, 2, \ldots, n, \end{aligned} \tag{2.1}$$

where $x = (x_1, x_2, \ldots, x_n)^{\mathrm{T}}$, x_i is the state variable of the ith unit; $\alpha(\cdot) = \mathrm{diag}(\alpha_1(\cdot), \alpha_2(\cdot), \ldots, \alpha_n(\cdot))$, $\alpha_i(\cdot) > 0$ is the amplification function; $D(t) = \mathrm{diag}(d_1(t), d_2(t), \ldots, d_n(t))$, $d_i(t)$ is the self-inhibition of the ith neuron; $A(t) = (a_{ij}(t))_{n\times n}$, $a_{ij}(t)$ is the connection strength of the jth neuron on the ith neuron; $B(t) = (b_{ij}(t))_{n\times n}$, $C(t) = (c_{ij}(t))_{n\times n}$, $b_{ij}(t)$ and $c_{ij}(t)$ are the delayed feedbacks of the jth neuron on the ith neuron, with time-varying delays and distributed delays, respectively; $f(x(t)) = (f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t)))^{\mathrm{T}}$, $f_i(x_i(t))$ is the neuron input–output activation of the ith neuron; $I(t) = (I_1(t), I_2(t), \ldots, I_n(t))^{\mathrm{T}}$, $I_i(t)$ is the external input to the ith neuron.

Throughout this paper, we always assume that all the coefficients $d_i(t), a_{ij}(t), b_{ij}(t), c_{ij}(t), I_i(t)$ are bounded continuous functions; the amplification function $\alpha_i(\cdot)$ is bounded differentiable