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Stochastic exponential synchronization of memristive neural networks with time-varying delays via quantized control

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ABSTRACT

This paper focuses on stochastic exponential synchronization of delayed memristive neural networks (MNNs) by the aid of systems with interval parameters which are established by using the concept of Filippov solution. New intermittent controller and adaptive controller with logarithmic quantization are structured to deal with the difficulties induced by time-varying delays, interval parameters as well as stochastic perturbations, simultaneously. Moreover, not only control cost can be reduced but also communication channels and bandwidth are saved by using these controllers. Based on novel Lyapunov functions and new analytical methods, several synchronization criteria are established to realize the exponential synchronization of MNNs with stochastic perturbations via intermittent control and adaptive control with or without logarithmic quantization. Finally, numerical simulations are offered to substantiate our theoretical results.

1. Introduction

The fundamental circuit elements, capacitor, resistor, and inductor are well known. They can be used to represent the relationships between essential electrical quantities which include current, voltage, flux and charge. Concretely, the capacitor describes a relationship between charge and voltage, resistor represents a relationship between current and voltage, and inductor stands for a relation between flux and current, respectively. It should be noted that the relation of flux and charge is not known until 1971. In this year, Chua (1971) predicted that there should be the fourth fundamental element, which is named memristor. Memristor represents a relationship between flux and charge, which cannot be replaced by any mixture of capacitors, resistors and inductors. In 2008, Chua's predictability was confirmed by the Hewlett–Packard research team (Strukov, Snider, Stewart, & Williams, 2008). This research team built a practical memristor device. After that, memristors receive more and more attention (Anbuvithya, Mathiyalagan, Sakthivel, & Prakash, 2016; Wang, Shen, Yin, & Zhang, 2015; Yang, Guo, & Wang, 2015).

Recently, dynamical behaviors of MNNs have garnered widescale attention since this class of neural networks (NNs) can be applied to emulate the human brain (Itoh & Chua, 2009; Thomas, 2013). Especially, synchronization of MNNs has been extensively investigated thanks to their potential applications in secure communications, image encryption, information science and biological systems (Mathiyalagan, Anbuvithya, Sakthivel, Park, & Prakash, 2016; Nana, Woafo, & Domngang, 2009; Sakthivel, Anbuvithya, Mathiyalagan, Ma, & Prakash, 2016; Wen, Huang, Zeng, Chen, & Li, 2015), and so on. Particularly, exponential synchronization of MNNs has become a hot research topic, and fruitful results have been achieved (Bao, Park, & Cao, 2015; Wang & Shen, 2014; Wen, Bao, Zeng, Chen, & Huang, 2013; Yang, Cao, & Yu, 2014; Zhang, Li, Huang, & He, 2015). For example, exponential synchronization of delayed memristive Cohen–Grossberg NNs was considered by structuring nonlinear transformation in Yang et al. (2014). Exponential synchronization of coupled memristive recurrent NNs was investigated in Zhang et al. (2015), where impulses with and without delay were considered for modeling the coupled NNs simultaneously. On the other hand, researchers have proposed various control methods to realize synchronization of MNNs and other chaotic systems, such as state feedback control (Wang & Shen, 2015; Yang & Ho, 2016), impulsive control (Li, Feng, & Huang, 2008; Li & Song, 2017), intermittent control (Huang, Li, Yu, & Chen, 2009; Zhang & Shen, 2015), pinning control (Wang, Wu, Huang, Ren, & Wu, 2016), and adaptive control (Song & Sun, 2017; Yang, Cao, & Liang, 2017). Note that intermittent control and adaptive control are efficient and economical in reducing the control cost among the above-mentioned types of control technique. Therefore, intermittent control and adaptive control have aroused

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widespread interests of researchers and witnessed effectiveness in synchronizing MNNs, respectively.

Note that NNs are always in an external noisy environment and therefore their behaviors maybe disturbed by uncertain perturbations. Stochastic dynamical systems are often used to describe the state of systems which are subjected to a changing environment. In fact, stochastic systems have attracted increasing attention due to their extensive application in economic systems, network control systems and modeling production systems (Bao, Park, & Cao, 2016; Huang, Li, Duan, & Starzyk, 2012; Li, Fang, & Li, 2017; Li, Yu, & Huang, 2014; Tang, Gao, & Kurths, 2014; Wu & Yang, 2016; Yang & Cao, 2009; Yang, Zhu, & Huang, 2011; Zhang, Li, Huang, & Huang, 2018). Particularly, asymptotic synchronization of complex-variable chaotic systems was considered in Wu and Yang (2016), where node systems presented nonidentical nonlinear dynamic behaviors and subjected to stochastic perturbations. In Li et al. (2017), the authors investigated exponential stabilization of delayed MNNs with stochastic disturbance via intermittent adaptive control. The authors focused on fixed-time synchronization of complex networks with nonidentical nodes and stochastic noise perturbations in Zhang et al. (2018). The limit speed of switching of amplifiers, memory effects, finite transmission velocity, etc. lead to inevitable presences of time delays in various fields such as engineering, physics, and biology. Hence, many results considered dynamical systems with time delays such as Bao et al. (2015, 2016), Huang, Chan, Huang, and Cao (2007) Huang et al. (2012, 2009), Li and Song (2017), Song and Sun (2017), Wang and Shen (2014, 2015), Wen et al. (2013), Yang and Cao (2009), Yang et al. (2017, 2014), Yang and Ho (2016), Yang et al. (2011), Zhang et al. (2015), and Zhang and Shen (2015).

On the other hand, the transmission of signals is usually limited by capacity and bandwidth of communication channels in practice. To improve the efficiency of communication, an effective method is quantizing the control signals before they are transmitted. The quantization problems have attracted considerable attention in recent years (Brockett & Liberzon, 2000; Cheng, Chang, Park, Li, & Wang, 2018; Song, Li, Li, & Lu, 2016; Tian, Yue, & Peng, 2008; Wan, Cao, & Wen, 2017; Xu et al., 2017). Especially, with the help of quantized control, stabilization of nonlinear discrete-time systems was considered in Song et al. (2016). Finite-time synchronization of coupled NNs was investigated via aperiodically intermittent pinning controllers with logarithmic quantization in Xu et al. (2017). There is no doubt that quantized control techniques can make full use of transmission capacity of the network and reduce channel blocking. Moreover, consider the quantized intermittent control and quantized adaptive control will further reduce the control cost. Note that there is no paper focus on exponential synchronization of MNNs via quantized intermittent control or quantized adaptive control in the open literature. This gap will be filled in the present paper, which is quite challenging.

Motivated by the above discussions, stochastic exponential synchronization for MNNs with time-varying delays is investigated in this paper. The main contributions are summarized below: (1) New analytical methods are adopted to study synchronization of MNNs by the aid of systems with interval parameters which are established by using the concept of Filippov solution; (2) New quantized intermittent controller and quantized adaptive controller are designed, which can make full use of transmission capacity of the network and reduce control cost; (3) By utilizing Lyapunov functional methods and stochastic analysis techniques, some exponential synchronization criteria of MNNs are obtained by applying quantized intermittent control and quantized adaptive control; (4) As special cases, some exponential synchronization criteria of MNNs are also derived by using controllers without logarithmic quantization.

The reminder of this technical correspondence is proposed as follows. The model of MNNs with stochastic perturbations is presented in following section. This section also presents some definitions and necessary assumptions which are important to derive our main results. In Section 3, exponential synchronization of the MNNs is studied via quantized intermittent controllers. In Section 4, exponential synchronization of the MNNs is studied via quantized adaptive controllers. Then, numerical simulations are carried out to substantiate the effectiveness of theoretical results in Section 5. In Section 6, the conclusion is given.

2. Model description and some preliminaries

Notations: The standard notations are used throughout this paper. $\mathbb{R}$ stands for the set of real numbers, $\mathbb{R}^n$ denotes the n-dimensional space, and $x = (x_1, x_2, \dots, x_n)^T$ is a vector of n-dimension. For matrix $A = (a_{ij})_{n\times n}$, $|A| = (|a_{ij}|)_{n\times n}$, $\mathrm{diag}(l_1, l_2, \dots, l_n)$ stands for a $n \times n$ diagonal matrix, I_n denotes the $n \times n$ identity matrix, and $\|\cdot\|$ represents the standard 2-norm of a vector or a matrix. Moreover, let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\ge 0}, \mathcal{P})$ be a complete probability space with filtration $\{\mathcal{F}_t\}_{t\ge 0}$ satisfying the usual conditions (i.e. the filtration contains all $\mathcal{P}$-null sets and is right continuous). $\mathbb{E}[\cdot]$ stands for mathematical expectation operator. $\overline{co}[E]$ is the closure of the convex hull of the set E.

Consider a model of delayed MNN which can be described as follows:

$$\mathrm{d}x_i(t) = [-c_i x_i(t) + \sum_{j=1}^{n} a_{ij}(x_i(t))f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij}(x_i(t))f_j(x_j(t-\tau_j(t)))]\mathrm{d}t, \tag{1}$$

where $i = 1, 2, \dots, n$, $x_i(t) \in \mathbb{R}$ denotes the voltage applied on the capacitor C_i, $\tau_i(t)$ is the time-varying delay; $c_i > 0$ stands for the rate with which the ith neuron will reset its potential to the resting state; $f(x(t)) = (f_1(x_1(t)), \dots, f_n(x_n(t)))^T$ is the neuron feedback function; $a_{ij}(x_i(t))$ and $b_{ij}(x_i(t))$ denote the non-delayed and delayed memristive synaptic connection weights, respectively, and

$$a_{ij}(x_i(t)) = \frac{W_{aij}(x_j(t))}{C_i}\mathrm{sgn}_{ij},\ b_{ij}(x_i(t)) = \frac{W_{bij}(x_j(t))}{C_i}\mathrm{sgn}_{ij},$$

$$\mathrm{sgn}_{ij} = \begin{cases} 1, & i = j, \\ -1, & i \neq j, \end{cases}$$

where $W_{aij}(x_j(t))$ and $W_{bij}(x_j(t))$ denote the memductances of memristors M_{aij} and M_{bij}, respectively. M_{aij} represents the memristor between $f_j(x_j(t))$ and $x_i(t)$, M_{bij} represents the memristor between $f_j(x_j(t-\tau_j(t)))$ and $x_i(t)$.

Based on the feature of memristor and the current–voltage characteristics, the following mathematical model of the memristance is considered:

$$a_{ij}(x_i(t)) = \begin{cases} \hat{a}_{ij}, & |x_i(t)| \le T_i, \\ \check{a}_{ij}, & |x_i(t)| > T_i, \end{cases} \tag{2}$$

$$b_{ij}(x_i(t)) = \begin{cases} \hat{b}_{ij}, & |x_i(t)| \le T_i, \\ \check{b}_{ij}, & |x_i(t)| > T_i, \end{cases} \tag{3}$$

where $T_i > 0$ is switching jump, $\hat{a}_{ij}, \check{a}_{ij}, \hat{b}_{ij}, \check{b}_{ij}, i, j = 1, 2, \dots, n$, are constants. The initial conditions of (1) are $x_i(t) = \phi_i(t) \in C([-\tau, 0], \mathbb{R}), i = 1, 2, \dots, n$.

The controlled response system is described as follows:

$$\mathrm{d}y_i(t) = [-c_i y_i(t) + \sum_{j=1}^{n} a_{ij}(y_i(t))f_j(y_j(t)) + \sum_{j=1}^{n} b_{ij}(y_i(t))f_j(y_j(t-\tau_j(t))) + r_i(t)]\mathrm{d}t + h_i(t, z_i(t), z_i(t-\tau_i(t)))\mathrm{d}\omega_i(t), \tag{4}$$