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Boundedness and global robust stability analysis of delayed complex-valued neural networks with interval parameter uncertainties ☆

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ABSTRACT

In this paper, the boundedness and robust stability for a class of delayed complex-valued neural networks with interval parameter uncertainties are investigated. By using Homomorphic mapping theorem, Lyapunov method and inequality techniques, sufficient condition to guarantee the boundedness of networks and the existence, uniqueness and global robust stability of equilibrium point is derived for the considered uncertain neural networks. The obtained robust stability criterion is expressed in complex-valued LMI, which can be calculated numerically using YALMIP with solver of SDPT3 in MATLAB. An example with simulations is supplied to show the applicability and advantages of the acquired result.

1. Introduction

In the past three decades, neural networks have been paid a great deal of attention in many engineering areas such as signal processing, image processing, pattern recognition, associative memory and optimization (Choi, Ahn, Karimi, & Lim, 2017; Huang, Li, Duan, & Starzyk, 2012; Zeng & Zheng, 2013). In some of these applications, it is required that the designed neural networks have a unique equilibrium point that is globally stable (Sun & Feng, 2003). When implementing neural networks, time delays are unavoidably encountered due to finite switching speeds of the amplifiers, which can significantly affect the stability of networks (Ahn, Shi, & Wu, 2015; Chen & Rong, 2003). Thus, it is necessary to study the stability of the delayed neural networks. On the other hand, the stability may be destroyed by some unavoidable uncertainty caused by the existence of modeling errors, external disturbance and parameter fluctuation. As a result, it requires that the designed neural networks have robustness (Arik, 2014a). Therefore, robust stability analysis of neural networks in the presence of time delays and parameter uncertainties is an important problem. In the recent literature, some useful results concerning the existence, uniqueness and global robust stability of equilibrium point for interval neural networks with time delays and parameter uncertainties have been reported, for example, see Arik (2014a), Arik (2014b), Cao, Huang, and Qu (2005), Cao, Li, and Han (2006), Cao and Wang (2005), Cui, Zhao, and Guo (2009), Ensari and Arik (2010), Faydasicok and Arik (2012), Faydasicok and Arik (2013), Ozcan (2011), Ozcan and Arik (2014), Samli (2015), Samli and Yucel (2015), Shao, Huang, and Wang (2011), Shao, Huang, and Wang (2012), Shao, Huang, and Zhou (2010), Singh (2007), Song and Cao (2007), Sun and Feng (2003), Yuan and Li (2010), Yucel (2015), Zhao and Zhu (2010) and the references therein. In Ozcan and Arik (2014) and Sun and Feng (2003), the interval neural networks with constant delays were considered, and several algebraic inequality criteria were obtained to ensure global robust stability of unique equilibrium point. In Cao and Wang (2005), a main sufficient condition in matrix norm inequality form was given to ensure global robust stability of unique equilibrium point for interval neural networks with constant delay. Further, some criteria in positive (negative) definite matrix form were provided to guarantee global robust stability of unique equilibrium point for interval neural networks with constant delay (Arik, 2014b; Cao et al., 2005, 2006; Cao & Wang, 2005; Ensari & Arik, 2010; Faydasicok & Arik, 2012, 2013; Rong, 2005; Samli,

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2015; Zhao & Zhu, 2010), or for interval neural networks with time-varying delays (Cui et al., 2009; Samli & Yucel, 2015; Shao et al., 2011, 2012, 2010), where elements of the matrices in criteria are the norm of connection weight matrices. From these criteria given, the sign of connection weight matrices in neural networks was ignored owing to the norm form of connection weight matrices, which may lead to conservatism. Consequently, it is a meaningful work to reduce the conservatism of the criteria.

The neural networks mentioned above are called real-valued neural networks (RVNNs) because the state, activation function, external input and connection weight of networks are real values. As a generalization of RVNNs, complex-valued neural networks (CVNNs) have been extensively studied in recent years since CVNNs make it possible to solve some problems which cannot be solved with their real-valued counterparts. For example, the XOR problem and the detection of symmetry problem cannot be solved with a single real-valued neuron, but they can be solved with a single complex-valued neuron with the orthogonal decision boundaries, which reveals the potent computational power of complex-valued neurons (Jankowski, Lozowski, & Zurada, 1996). And there have been some researches on the stability of various CVNNs, for example, see Fang and Sun (2014), Hu and Wang (2012), Jankowski et al. (1996); Lee (2001), Liu and Chen (2016), Pan, Liu, and Xie (2015), Rao and Murthy (2008), Song, Yan, Zhao, and Liu (2016a), Song, Yan, Zhao, and Liu (2016b), Song, Zhao, and Liu (2015a), Song, Zhao, and Liu (2015b), Zhang, Li, and Huang (2014), Zhang, Lin, and Chen (2014), Zhang, Liu, Chen, Guo, and Zhou (2017) and Zhou and Song (2013). In Jankowski et al. (1996), a CVNNs was proposed, and the stability of the network was discussed under the assumption that the weight matrix was Hermitian with nonnegative diagonal entries. Immediately following, Lee (2001) weakened the assumption of weight matrix in Jankowski et al. (1996). In Fang and Sun (2014), Hu and Wang (2012), Liu and Chen (2016), Zhang, Lin et al. (2014), Zhang et al. (2017) and Zhou and Song (2013), the researchers investigated the asymptotical stability and exponential stability of CVNNs with constant delay. The CVNNs with time-varying delays were considered and some sufficient conditions for stability of a unique equilibrium were derived by Pan et al. (2015) and Song et al. (2015a). Rao and Murthy (2008) and Song et al. (2015b) also studied the stability of discrete-time CVNNs. Furthermore, impulsive effect on stability of CVNNs with time delays was considered by Song et al. (2016a, 2016b) and Tan, Tang, Yang, and Liu (2017).

Recently, the robust stability problem for delayed CVNNs with parameter uncertainties has been considered. For example, see Gong, Liang, Zhang, and Cao (2016), Tan et al. (2017), and Zhang, Li, et al. (2014). In Zhang, Li, et al. (2014), sufficient condition to insure the existence, uniqueness and global robust stability of equilibrium point for delayed CVNNs with interval parameter uncertainties was established under the assumptions that activation functions can be separated into its real part function and imaginary part function, and real part function and imaginary part function satisfy Lipschitz conditions and their partial derivatives are continuous and bounded. Then, when complex-valued activation functions are bounded and satisfy Lipschitz continuity condition in the complex domain, sufficient conditions to ensure the existence and global robust stability of equilibrium point for delayed CVNNs with interval parameter uncertainties were provided. From the given criteria, the sign of the entries of the connection matrices was not considered since the obtained sufficient conditions were expressed by the norm of the matrix, which may lead to conservatism. In Gong et al. (2016), based on the nonlinear measure method and the matrix inequality techniques, the robust stability criterion for CVNNs with constant delays and parameter uncertainties was obtained under the assumption that activation functions can be separated into its real and imaginary parts. As pointed out in Song et al. (2016b), there are two problems with this approach of separating activation functions. One is that the dimension will double increase when CVNNs are transformed into real-valued systems, which can lead to the difficulties in analyzing stability. The other is that this approach needs to explicitly separate the complex-valued activation functions into the real and imaginary parts. In general, this separation in an analytical form is not always expressible.

Motivated by the above deliberation, the purpose of this paper is to present a new sufficient condition for the global robust stability of delayed CVNNs with interval parameter uncertainties. By giving a numerical example, we will also compare our result with the previous relevant robust stability results derived in the literature. The main contributions of this paper are listed as follows. (1) The considered CVNNs are not separated into their real and imaginary parts. (2) The established stability criterion depends on both the lower and upper bounds of intervals for interval parameter uncertainties. (3) The sign of the entries of the connection weight matrices was not ignored.

Notations: Throughout this paper, I represents the unitary matrix with appropriate dimensions; $\mathbb{R}^n$ and $\mathbb{C}^n$ denote, respectively, the set of all n-dimensional real-valued vectors and complex-valued vectors. $\mathbb{R}^{n\times m}$ and $\mathbb{C}^{n\times m}$ denote, respectively, the set of all $n \times m$ real-valued matrices and complex-valued matrices. A^* shows the complex conjugate transpose of complex-valued matrix A. $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ are defined as the largest and the smallest eigenvalue of Hermitian matrix P, respectively. The subscript T denotes the matrix transposition. The notation $X > Y$ means that X and Y are Hermitian matrices, and that $X - Y$ is positive definite. $|a|$ denotes the module of complex number $a \in \mathbb{C}$, and $\|z\|$ denotes the norm of $z \in \mathbb{C}^n$, i.e., $\|z\| = \sqrt{z^*z}$. If $A \in \mathbb{C}^{n\times n}$, denote by $\|A\|$ its norm. For $a, b \in \mathbb{C}$, $a \preceq b$ denotes $a_1 \le b_1$ and $a_2 \le b_2$, where $a = a_1 + a_2 i$ and $b = b_1 + b_2 i$. For $A, B \in \mathbb{C}^{n\times n}$, $A \preceq B$ if and only if $a_{ij} \preceq b_{ij}$ for $i, j = 1, 2, \ldots, n$, where $A = (a_{ij})_{n\times n}$ and $B = (b_{ij})_{n\times n}$. In addition, the notation $\star$ always denotes the conjugate transpose of a suitable block in a Hermitian matrix.

2. Model description and preliminaries

In this paper, we consider the following delayed CVNNs with interval parameter uncertainties

$$\dot{z}(t) = -Cz(t) + Af(z(t)) + Bf(z(t-\tau)) + J \tag{1}$$

for $t \ge 0$, where $z(t) = (z_1(t), z_2(t), \ldots, z_n(t))^T \in \mathbb{C}^n$, $z_i(t)$ is the state of the ith neuron at time t; τ corresponds to the transmission delay; $f(z(t)) = (f_1(z_1(t)), f_2(z_2(t)), \ldots, f_n(z_n(t)))^T \in \mathbb{C}^n$ is the vector-valued activation function whose elements consist of complex-valued nonlinear functions; $C = \mathrm{diag}\{c_1, c_2, \ldots, c_n\} \in \mathbb{R}^{n\times n}$ is the self-feedback connection weight matrix, where $c_i > 0$; $A \in \mathbb{C}^{n\times n}$ is the connection weight matrix, $B \in \mathbb{C}^{n\times n}$ is the delayed connection weight matrix; $J \in \mathbb{C}^n$ is the input vector. The initial condition associated with (1) is given by

$$z(s) = \phi(s), \quad s \in [-\tau, 0], \tag{2}$$

where $\phi(s) \in \mathbb{C}^n$ is continuous in $[-\tau, 0]$.

Throughout this paper, we make the following assumptions:

Assumption 1. For any $i \in \{1, 2, \ldots, n\}$, there exists a positive diagonal matrix $L = \mathrm{diag}\{l_1, l_2, \ldots, l_n\}$ such that

$$|f_i(\alpha_1) - f_i(\alpha_2)| \le l_i|\alpha_1 - \alpha_2|$$

for all $\alpha_1, \alpha_2 \in \mathbb{C}$.