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Robust generalized Mittag-Leffler synchronization of fractional order neural networks with discontinuous activation and impulses ☆

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ABSTRACT

Fractional order system is playing an increasingly important role in terms of both theory and applications. In this paper we investigate the global existence of Filippov solutions and the robust generalized Mittag-Leffler synchronization of fractional order neural networks with discontinuous activation and impulses. By means of growth conditions, differential inclusions and generalized Gronwall inequality, a sufficient condition for the existence of Filippov solution is obtained. Then, sufficient criteria are given for the robust generalized Mittag-Leffler synchronization between discontinuous activation function of impulsive fractional order neural network systems with (or without) parameter uncertainties, via a delayed feedback controller and M-Matrix theory. Finally, four numerical simulations demonstrate the effectiveness of our main results.

1. Introduction

In recent years, fractional order dynamical system has aroused interest of many researchers in the field of nonlinear science and technology. Fractional-order calculus, which generalized the classical calculus developed in the 17th century (Kilbas, Srivastava, & Trujillo, 2006; Podlubny, 1999). Fractional calculus investigates primarily the properties of derivatives and integrals of non-integer order. In particular, the differential equations involving fractional derivatives have important geometric interpretations. For this reason, fractional calculus is currently a rapidly growing field, in terms of both theory and applications to real world problem. More precisely, fractional calculus has been applied in various branches of science and engineering, including electromagnetic waves (Heaviside, 1971) and bioengineering (Magin & Ovadia, 2008). Compared to integer order calculus, fractional order one has infinite memory and more degrees of freedom (Chen, Jiao, Wu, & Wang, 2010; Chen, Ye, & Sun, 2010). Moreover, fractional order is also said to be "more authentic" (Hilfer, 2000). Nowadays, the dynamical system of synchronization or stability of fractional order neural networks was found to play an important role in applications, such as information theory, pattern recognition, cryptography or secure communication (Li & Cao, 2017a; Milanovic & Zaghloul, 1996; Ren, Cao, & Cao, 2015; Song & Cao, 2014; Yang et al., 2012).

Since the formulation of drive–response synchronization concept in 1990s by Carroll and Pecora, which means dynamical behaviors of a coupled system that realizes convergence to the matching spatial state, has become an important research topic in various areas. Still now, there are numerous types of synchronization concepts known in the literature, including complete synchronization (Ding, Shen, & Wang, 2016; Lu, Wang, Cao, Ho & Kurths, 2012; Yang & Cao, 2014), anti-synchronization (El-Dessoky, 2010), lag synchronization (Zhang, Lv, & Li, 2017), phase synchronization (Rosenblum, Pikovsky, & Kurths, 1996) and others. However, few practical network systems can be synchronized directly. To address this problem, several control schemes have been introduced, such as feedback control (Cao & Li, 2017; Li, Cao, Alsaedi, & Alsaadi, 2017; Li, Zhang, & Song, 2017), linear feedback control (Xiao, Zhong, Li, & Xu, 2016), observer based control (Jiang, Tang, & Chen, 2006) and impulsive control (Li & Song, 2017; Yang & Cao, 2007).

Generally, time delay of the signal between the driver and response system is unavoidable because of the network traffic congestion as well as finite switching speed of signal transmission over the links, which may lead to instability, chaos, oscillation or other performance of network models (Li, Cao et al., 2017; Li, Zhang et al., 2017). Moreover, time delays are more complicated compared to other networks (Chen, Chai, Wu, & Ma, 2013; Li & Wu, 2016; Zhu

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& Cao, 2011). Impulses, i.e., abrupt changes in state at certain times also affect the stability of the systems (Li, Bohner, & Wang, 2015; Li & Cao, 2017b; Li, Cao et al., 2017; Li, Zhang et al., 2017; Stamova, Stamov, & Li, 2014). Generally, impulsive systems belong to two major types: first is the constant impulsive system, while the other one is time varying impulsive system. Due to measurement errors, parameter fluctuations as well as external disturbance, parameter uncertainty is unavoidable, which has important effects on the stability and synchronization capability of most real world dynamical systems. Additionally, the main application of this dynamical problem is used to secure communication, only if the drive and response systems realize synchronization can the transmitted signal be fingered out. Therefore, it is necessary to study fractional order complete synchronization of neural networks with discontinuous activations. Firstly, the global convergence of general neural networks with discontinuous activations were considered in Forti and Nistri (2003), while Forti, Nistri, and Papini (2005) discussed the infinite gain of neural discontinuous activations. Besides, these activations are mainly applied to systems oscillating under earthquake, dry friction, power circuits and so on. Several results with respect to synchronization of discontinuous neural networks have been reported in the literature (Liu & Cao, 2009; Liu, Liu, & Xie, 2012; Liu, Park, Jiang, & Cao, 2014; Lu & Chen, 2005). On the other hand, Wang, Shen, and Sheng (2016), some parameter uncertain models of integer order delayed neural networks with discontinuous activations are discussed, while Ding et al. (2016) investigated Mittag-Leffler synchronization of neural networks with discontinuous activation functions by using M-matrix theory and non smooth analysis. However, there are few results of synchronization of fractional neural networks with discontinuous activation. To our best of knowledge, there are no results published in robust generalized Mittag-Leffler synchronization of delayed neural network systems (GMSDNNs) with (or without) parameter uncertainties. This model is more general and can be extended beyond the study of integer order discontinuous dynamical systems.

Inspired by the above analysis and discussions, our main aim in this paper, is to study the generalized Mittag-Leffler synchronization of delayed fractional order neural networks(GMSDNNs) with discontinuous activations. The crucial novelty of this paper is further summarized as follows:

- In the sense of Caputo fractional order derivative of $0 < \alpha < 1$, based on the growth condition and non smooth analysis, we have proved the global existence of Filippov solution.
- A delayed feedback controller is designed which includes the constant time delay terms and discontinuous term.
- By means of M-matrix theory, Lyapunov stability theory and proposed discontinuous control scheme, the algebraic sufficient condition for generalized Mittag-Leffler synchronization is addressed, and we improved the fractional order continuous activation synchronization methods. Moreover, an important feature presents in our paper is that the improved result is still true for integer order robust exponential synchronization of delayed neural networks with discontinuous (continuous) activations with impulses.

The rest of the paper is organized as follows. In Section 2, some basic definitions and preliminaries are given including the problem formulation are introduced. In Section 3, the existence of Filippov solution is provided and derived the sufficient criteria for the robust generalized Mittag-Leffler synchronization between drive and response neural network systems. Section 4 considers the four numerical examples to validate the theoretical obtained results, conclusions are drawn in Section 5.

2. Model description and preliminaries

Notations: Throughout this paper, $\mathbb{R}$ is the space of real number, $\mathbb{N}_+$ is the set of positive integers and $\mathbb{C}$ is the space of complex numbers. For a vector $x \in \mathbb{R}^n$, we shall use the norm $\|x\| = \|.\|_1 = \sum_{i=1}^{n}|x_i|$. The signum function applied for a vector $\mathrm{sgn}(x) = [\mathrm{sgn}(x_1), \mathrm{sgn}(x_2), \ldots, \mathrm{sgn}(x_n)]^T$ is given by

$$\mathrm{sgn}(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0. \end{cases}$$

Also, $\mathbb{R}^{n\times n}$ denotes the set of all $n \times n$ real matrices. For a square matrix $A = (a_{ij})_{n\times n} \in \mathbb{R}^{n\times n}$, we consider the absolute value given by the formula $|A| = (|a_{ij}|)_{n\times n} \in \mathbb{R}^{n\times n}$. In addition $C^n([t_0, +\infty), \mathbb{R})$ denotes the space consisting of n-order continuous differentiable functions from $[t_0, +\infty)$ into $\mathbb{R}$.

For our further presentation and convenience, we set the following notations:

$$\begin{aligned}
\underline{D} &= diag(\underline{d}_1, \underline{d}_2, \ldots, \underline{d}_n),\ \hat{a}_{ij} = \max\{|\underline{a}_{ij}|, |\bar{a}_{ij}|\},\\
\hat{b}_{ij} &= \max\{|\underline{b}_{ij}|, |\bar{b}_{ij}|\},\\
K_1 &= diag(\kappa_{1,1},\ \kappa_{1,2}, \ldots, \kappa_{1,n}),\ K_2 = diag(\kappa_{2,1}, \kappa_{2,2}, \ldots, \kappa_{2,n}),\\
K_3 &= diag(\kappa_{3,1},\ \kappa_{3,2},\ \ldots, \kappa_{3,n}),\\
E_1 &= (\hat{a}_{ij}p_j)_{n\times n},\ \ E_2 = \Big((\hat{a}_{ij} + \hat{b}_{ij})p_j\Big)_{n\times n},\ E_3 = (|a_{ij}|p_j)_{n\times n},\\
E_4 &= \Big((|a_{ij}| + |b_{ij}|)p_j\Big)_{n\times n},\\
F_1 &= diag\Big\{\sum_{j=1}^{n}\hat{b}_{1j}p_j, \sum_{j=1}^{n}\hat{b}_{2j}p_j, \ldots, \sum_{j=1}^{n}\hat{b}_{nj}p_j\Big\},\\
F_2 &= diag\Big\{\sum_{j=1}^{n}(\hat{a}_{1j} + \hat{b}_{1j})q_j, \sum_{j=1}^{n}(\hat{a}_{2j} + \hat{b}_{2j})q_j, \ldots,\\
&\qquad \sum_{j=1}^{n}(\hat{a}_{nj} + \hat{b}_{nj})q_j\Big\},\\
F_3 &= diag\Big\{\sum_{j=1}^{n}\hat{a}_{1j}q_j, \sum_{j=1}^{n}\hat{a}_{2j}q_j, \ldots, \sum_{j=1}^{n}\hat{a}_{nj}q_j\Big\},\\
M_1 &= diag\Big\{\sum_{j=1}^{n}|b_{1j}|p_j, \sum_{j=1}^{n}|b_{2j}|p_j, \ldots \sum_{j=1}^{n}|b_{nj}|p_j\Big\},\\
M_2 &= diag\Big\{\sum_{j=1}^{n}(|a_{1j}| + |b_{1j}|)q_j, \sum_{j=1}^{n}(|a_{2j}| + |b_{2j}|)q_j, \ldots,\\
&\qquad \sum_{j=1}^{n}(|a_{nj}| + |b_{nj}|)q_j\Big\}.
\end{aligned}$$

In this section we recalled some key definitions, assumption and some basic lemmas.

Definition 2.1 (*Kilbas, Srivastava, & Trujillo, 2006 & Podlubny, 1999*). The Caputo fractional-order derivative of order α for a function $x(t) \in C^n([t_0, +\infty))$ is defined as

$$D^{\alpha}x(t) = \frac{1}{\Gamma(n-\alpha)}\int_{t_0}^{t}\frac{x^n(s)}{(t-s)^{\alpha-n+1}}\,\mathrm{d}s,$$

where $t \geq t_0$ and n is the positive integer such that $n - 1 < \alpha < n$.

Particularly, when $0 < \alpha < 1$,

$$D^{\alpha}x(t) = \frac{1}{\Gamma(1-\alpha)}\int_{t_0}^{t}\frac{x'(s)}{(t-s)^{\alpha}}\,\mathrm{d}s.$$