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Neural robust stabilization via event-triggering mechanism and adaptive learning technique[☆]

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ABSTRACT

The robust control synthesis of continuous-time nonlinear systems with uncertain term is investigated via event-triggering mechanism and adaptive critic learning technique. We mainly focus on combining the event-triggering mechanism with adaptive critic designs, so as to solve the nonlinear robust control problem. This can not only make better use of computation and communication resources, but also conduct controller design from the view of intelligent optimization. Through theoretical analysis, the nonlinear robust stabilization can be achieved by obtaining an event-triggered optimal control law of the nominal system with a newly defined cost function and a certain triggering condition. The adaptive critic technique is employed to facilitate the event-triggered control design, where a neural network is introduced as an approximator of the learning phase. The performance of the event-triggered robust control scheme is validated via simulation studies and comparisons. The present method extends the application domain of both event-triggered control and adaptive critic control to nonlinear systems possessing dynamical uncertainties.

1. Introduction

The robustness of control system is a significant topic during the development of both control theory research and control engineering applications, since the common existences of model uncertainties, exogenous disturbances or other changes. These phenomena may result in bad control performance if they are not handled appropriately. Hence, we can apparently observe that the importance of coping with robust stabilization problem has been recognized by control scientists for many years (see, e.g., Lin, Brand, and Sun (1992) and the references therein). We consider the robust synthesis based on optimal control method in this paper. When discussing and implementing the nonlinear optimal control problem, we always encounter a fundamental issue, namely, solving the Hamilton–Jacobi–Bellman (HJB) equation. It is also computationally difficult to apply dynamic programming to solve complex optimal control problems because of the backward-in-time manner and the "curse of dimensionality". Based on neural networks, Werbos (1974, 1992) proposed adaptive/approximate dynamic programming (ADP) to solve optimal control problems forward-in-time. Then, some related results have been obtained such as in artificial intelligence (Lewis & Vrabie, 2009), automatic control (Dierks & Jagannathan, 2012; Heydari, 2014; Luo, Wu, Huang, & Liu, 2015; Mu & Wang, 2017; Yang, Liu, Wang, & Wei, 2014; Zhang, Qin, & Luo, 2014), operational research and so on. Among them, Dierks and Jagannathan (2012) proposed an online optimal control approach for nonlinear discrete-time systems with input-affine form and unknown internal dynamics by using the time-based policy update. Heydari (2014) revisited the ADP algorithm and developed some new convergence results of both the inner-loop and outer-loop iterations. Incidentally, the ADP technique is closely related to reinforcement learning (Lewis & Vrabie, 2009), one of whose fundamental algorithms is policy iteration (Abu-Khalaf & Lewis, 2005; Modares, Lewis, & Naghibi-Sistani, 2013; Vamvoudakis & Lewis, 2010). For instance, Modares et al. (2013) presented an adaptive control method that can converge to the optimal state feedback law for unknown continuous-time systems with input constraints through the policy iteration algorithm and an actor–critic structure.

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In 2011, Adhyaru, Kar, and Gopal (2011) proposed the HJB equation based offline optimal control method for robust stabilization of nonlinear systems. Then, Wang, Liu, and Li (2014) constructed an online design method for solving the nonlinear robust stabilization problem via the policy iteration algorithm and the results were extended to data-based robust optimal controller design with unknown dynamics (Wang, Liu, Zhang, & Zhao, 2016). In addition, Jiang and Jiang (2014) studied the robust ADP framework. Fan and Yang (2016) proposed an improved sliding mode control method by using the idea of ADP.

As indicated in many literature, the designed controller via time-driven manner always requires periodic transmitted data according to a fixed sampling period. However, the large number of transmitted data may bring in tremendous computation, which may lead to the reduction of control efficiency. Then, the event-triggered control formulation has recently acquired much attention (Liu & Jiang, 2015; Sahoo, Xu, & Jagannathan, 2016; Shi, Wang, & Lim, 2016). For example, Shi et al. (2016) stated that the sampled signal was transmitted according to a triggering condition other than a fixed time interval of the traditional time-triggered scheme. In other words, under this design mechanism, the control signal is updated once an event is triggered, and therefore, the computation can be significantly saved. By combining with neural network technique, Sahoo et al. (2016) studied the approximation-based event-triggered control for multi-input multi-output continuous-time unknown affine nonlinear systems, where the control input could be directly approximated via a neural network in the context of event-based transmission. The event-triggered manner has also been combined with the idea of ADP (Vamvoudakis, 2014; Wang, Mu, He, & Liu, 2016; Zhang, Zhao, & Zhu, 2017; Zhong, Ni, He, Xu, & Zhao, 2014). Vamvoudakis (2014) proposed a novel event-triggered adaptive optimal control method. Then, an event-triggered reinforcement learning strategy for nonlinear systems with unknown dynamics was proposed based on the measured input–output data (Zhong et al., 2014). Nevertheless, it is obvious to find that the system uncertainties are not always considered in the existing publications of ADP-based event-triggered control. That is to say, the lack of nonlinear robust stabilization through event-triggering framework and adaptive critic technique greatly motivates our research. Therefore, in this paper, inspired by Vamvoudakis (2014) and Wang et al. (2014), we extend the work of Wang et al. (2016) and revisit the nonlinear robust control problem via event-triggering mechanism and adaptive critic technique. Unlike Vamvoudakis (2014) and Wang et al. (2014), the effective event-triggered mechanism is employed to investigate the adaptive-critic-based nonlinear robust stabilization. First of all, the controlled problem is changed into designing an event-triggered optimal controller where a newly modified cost function is introduced to account for system uncertainties. Then, the event-triggered HJB equation and related optimal control law are obtained under the new mechanism. Next, an adaptive critic learning technique is used to cope with the HJB equation by building a critic network and implementing the weight training. We show that the approximate closed-form expression of the event-triggered optimal policy is available with the finally convergent weight vector, and accordingly, there is no necessity to further construct an action network. Moreover, the uniform ultimate boundedness (UUB) of the closed-loop system is proved based on the Lyapunov approach. In general, we have greatly extended the conference paper (Wang et al., 2016) both in theoretical analysis and in simulation verification. For one thing, the event-triggered robust adaptive critic control framework has been developed more carefully with clear flowchart description and rigorous stability proof. For another, the experimental simulation has been conducted more clearly with sufficient case studies and descriptions, which are different from that of Wang et al. (2016).

Table 1
Notation description.

Symbol	Meaning
$\mathbb{R}$	The set of all real numbers
$\mathbb{R}^n$	The Euclidean space of n-dimensional real vectors
$\mathbb{R}^{n\times m}$	The space of all $n \times m$ real matrices
$\\|\cdot\\|$	The vector norm or the matrix norm
I_n	An identity matrix with $n \times n$ dimension
$\lambda_{\min}(\cdot)$	The minimal eigenvalue of a matrix
$\lambda_{\max}(\cdot)$	The maximal eigenvalue of a matrix
Ω	A compact subset of $\mathbb{R}^n$
$\Psi(\Omega)$	The set of admissible control laws
i	The iteration index for weight updating
j	The sampling instant of event-triggering mechanism
T	The transpose operation
$\nabla(\cdot)$	The gradient operator, i.e., $\nabla(\cdot) \triangleq \partial(\cdot)/\partial x$

In summary, the major contributions of this paper compared with the existing references, especially Adhyaru et al. (2011), Fan and Yang (2016), Jiang and Jiang (2014), Modares et al. (2013), Sahoo et al. (2016), Vamvoudakis (2014), Vamvoudakis and Lewis (2010), Wang et al. (2014, 2016), Zhang et al. (2017) and Zhong et al. (2014), are given as follows. On one hand, the event-triggering formulation is taken into consideration for saving the communication resources of traditional time-based optimal control (Modares et al., 2013; Vamvoudakis & Lewis, 2010) and robust control designs (Adhyaru et al., 2011; Fan & Yang, 2016; Jiang & Jiang, 2014; Wang et al., 2014, 2016). On the other hand, compared with Sahoo et al. (2016), Vamvoudakis (2014), Zhang et al. (2017) and Zhong et al. (2014), the event-triggered robust control method is developed which extends the results of adaptive-critic-based optimal control and H_∞ control designs. Great efforts have been given to broaden the application scope of adaptive critic designs to nonlinear systems with uncertainties and meanwhile enhance the control efficiency of adaptive critic designs with event-triggering formulation. In Section 2, the robust control problem and the transformation background are stated briefly. In Section 3, we discuss the nonlinear robust stabilization under the event-triggering mechanism with the derivation of a new triggering condition. The adaptive neural implementation, closed-loop stability, and simulation verification are also provided in the content of Sections 3 and 4. In Section 5, concluding remarks and further discussions are given.

In this paper, the used notations are listed in Table 1.

2. Problem description and background

We study continuous-time nonlinear systems with the form

$$\dot{x}(t) = f(x(t)) + g(x(t))u(t) + \Delta f(x(t)), \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector and $u(t) \in \mathbb{R}^m$ is the control input, $f(\cdot)$ and $g(\cdot)$ are differentiable in their arguments with $f(0) = 0$, and $\Delta f(x(t))$ is the unknown perturbation. In this paper, we assume $\Delta f(x) = g(x)d(x)$ with $d(x) \in \mathbb{R}^m$ and let $x(0) = x_0$ be the initial state. Besides, assume that $d(x)$ with $d(0) = 0$ is upper bounded by a known function $d_M(x)$, i.e., $\|d(x)\| \le d_M(x)$ with $d_M(0) = 0$.

In order to cope with the robust control problem of system (1), we need to find a feedback control law $u(x)$ and render the closed-loop system to be asymptotically stable under the uncertainty $d(x)$. In this section, we provide the transformation solution for this problem by designing the optimal controller of the corresponding nominal system and introducing a suitable cost function.

Consider the nominal system

$$\dot{x}(t) = f(x(t)) + g(x(t))u(x(t)) \tag{2}$$