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Stability analysis for discrete-time stochastic memristive neural networks with both leakage and probabilistic delays☆

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ABSTRACT

This paper is concerned with the globally exponential stability problem for a class of discrete-time stochastic memristive neural networks (DSMNNs) with both leakage delays as well as probabilistic time-varying delays. For the probabilistic delays, a sequence of Bernoulli distributed random variables is utilized to determine within which intervals the time-varying delays fall at certain time instant. The sector-bounded activation function is considered in the addressed DSMNN. By taking into account the state-dependent characteristics of the network parameters and choosing an appropriate Lyapunov–Krasovskii functional, some sufficient conditions are established under which the underlying DSMNN is globally exponentially stable in the mean square. The derived conditions are made dependent on both the leakage and the probabilistic delays, and are therefore less conservative than the traditional delay-independent criteria. A simulation example is given to show the effectiveness of the proposed stability criterion.

1. Introduction

For decades, it has been generally recognized that the recurrent neural networks (RNNs) are capable of self-organizing, self-learning, nonlinear function approximation and fault tolerance. In fact, RNNs have been successfully applied in a variety of practical domains which include, but are not limited to, signal processing, control engineering, pattern recognition, image processing and combinatorial optimization. These applications are heavily dependent on the dynamic behaviors of the RNNs. In fact, the dynamics analysis problem for RNNs has been a hot topic of research receiving an ever-increasing research interest and a great many excellent results have been reported in the literature, see e.g. Liang, Gong, and Huang (2016), Liu, Wang, and Liu (2006), Liu, Wang, and Liu (2008), Shen, Wang, and Qiao (2017), Wang, Liu, and Liu (2005), Zhang, He, Jiang, Lin, and Wu (2017), Zhang, Tang, Wong, and Miao (2015) and the references therein. In particular, the global stability of RNNs is arguably the most desirable dynamic property that attracts a great deal of research attention and plays a vitally important role in practice such as optimization problems (Zhang, Wang, & Liu, 2014).

Since the first announcement from the HP Lab on the experimental prototyping of memristor, memristive devices have been widely investigated for their potential applications in non-volatile memories, logic devices, neuromorphic devices, and neuromorphic self-organized computation and learning, see Adamatzky and Chua (2013) and Strukov, Snider, Stewart, and Williams (2008) for more details. On the other hand, it is well known that NNs can be implemented by very large-scale integration and, in the implementation of NNs, it is natural to replace the resistors by the memristors in order to exploit the aforementioned advantages of memristors, and this gives rise to a new kind of neural networks, namely, memristive neural networks (MNNs). Actually, in the past few years, such MNNs have already been used in some application areas such as brain emulation, combinatorial optimization and knowledge acquisition, see e.g. Pedretti et al. (2017) and Pershin and Di Ventra (2010), where the dynamical behaviors (especially the global stability) of the MNNs form a critically important in the successes of the MNN applications. In this regard, along with the

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development/deployment of the MNNs, much research has been done on the stability analysis issues of MNNs and a number of excellent results have been available, see Wang, Ding, Huang, and Zhang (2016), Wang, Duan, Huang, Li, and Wang (2016), Wu and Zeng (2014) and Wu and Zeng (2017) for some recent results. For example, in Wang, Duan et al. (2016), some sufficient conditions have been established to guarantee the exponential stability of the MNNs with impulse effects. In Wu and Zeng (2014), the problem of Lagrange stability has been studied for MNNs with mixed time-delays by utilizing a nonsmooth analysis approach.

It is worth noticing that, up to now, most existing results concerning the stability analysis problem for MNNs have focused on the *continuous-time* and/or *deterministic* neural networks. Nevertheless, as is well known, in today's digital world, almost all signals are digitalized before and after being transmitted for the purpose of computer processing, and therefore signals in a *discrete-time* form are more practical than their continuous-time counterparts in simulating and applications of MNNs. Another factor that should be taken into account when modeling MNNs is the stochastic effects resulting from various causes such as the random synaptic transmissions released from the neurotransmitters in real nervous systems (Zhang, Xu, Zong, & Zou, 2009). As such, both discretization and stochasticity contribute much to the complexities of the dynamics analysis for MNNs (Han, Liu, & Yang, 2016), but the corresponding research on these two aspects is still in their infancy stage despite a great many results on discrete-time stochastic NNs (Liu et al., 2008), which is mainly due to the essential difficulties in handling the state-dependent-switching behaviors of MNNs.

Indeed, from the systems' perspective, a memristive neural network can be viewed as a state-dependent switching network (Wu & Zeng, 2014). So far, the stability problem has been extensively investigated for continuous-time MNNs and the frequently employed techniques are the Filippov differential equation with discontinuous right-hand side as well as the set-valued maps theory. A large number of results have been available on the establishment of various stability criteria for continuous-time MNNs with or without time-delays, see e.g. Wang, Duan et al. (2016), Wu and Zeng (2014) and Wu and Zeng (2017). Unfortunately, those techniques used to analyze continuous-time MNNs cannot be directly borrowed to deal with the discrete-time cases, and there is a lack of effective methods for handling the stochasticity. To this end, there are both theoretical and practical needs to examine the stability problem for *discrete-time stochastic* MNNs, which appears to be challenging because of the mathematical difficulties in coping with the state-dependent switches coupled with the discrete and stochastic fashions.

Time-delays are commonly encountered in the implementation of neural networks due to the finite speed of the transmission and switching of signals in a realistic biological system. It is now well recognized that time-delays are likely to be the sources of poor system performance including instability and oscillation. Ever since the introduction in Marcus and Westervelt (1989), delayed neural networks have been widely investigated and stability conditions have been attained in the literature by using a variety of techniques including descriptor model transformation approach (Fridman, 2001), integral inequality technique (Kwon, Park, Lee, Park, & Cha, 2013), matrix inequality technique (Song, Yan, Zhao, & Liu, 2016; Zhang & Han, 2014) and so on. In the context of MNNs, the aforementioned methods in combination with the set-valued mapping theory have been exploited to deal with the stability analysis of MNNs with time-delays (Wang, Duan et al., 2016; Wu & Zeng, 2014, 2017).

In spite of the considerable interest in the time-delays for RNNs, there are two classes of time-delays, namely, leakage and probabilistic delays, which have been relatively unexplored. On one hand, leakage delays are often encountered in the stabilizing negative feedback terms and, if not adequately handled, they would probably cause undesired dynamical behaviors or even destabilize the neural networks, see Chen, Fu, Liu, and Alsaadi (2017) and the references therein. On the other hand, the presence of time delay in neural networks might be randomly occurring due mainly to the random fluctuations of the synaptic voltage and temporal signals from transmitter release (Yue, Tian, Zhang, & Peng, 2009) and, therefore, a series of results have been reported on RNNs with probabilistic time-varying delays, see Sheng, Wang, Tian, and Alsaadi (2016), Song, Zhao, and Liu (2015), Yue et al. (2009) and the references therein. However, so far, the stability problem for *discrete-time stochastic* MNNs has not been thoroughly investigated yet, not to mention the case when the MNNs are also subject to both the leakage delay and the probabilistic time-varying delays.

Motivated by the above discussions, in this paper, we aim to deal with the exponential stability problem for DSMNNs with both leakage and probabilistic delays. It is worth noting that the Filippov differential approach and set-valued maps theory, which are effective in handling dynamics analysis issues for continuous-time MNNs, are no longer working for the DSMNNs to be addressed, and it is therefore necessary to develop new model/techniques. We first propose a new yet comprehensive model to account for the state-dependent behaviors of the network parameters in the discrete-time setting, and then employ an appropriate Lyapunov–Krasovskii functional to reflect the impacts from the leakage delay and the random characteristics of time-varying delays. Some conditions for the exponential stability of the addressed DSMNNs are obtained that have dependences on both the leakage and probabilistic delays in terms of their intensity and distribution laws. A simulation example is finally provided to show the usefulness and effectiveness of developed theoretical results.

The main contributions of this paper are highlighted as follows: *(1) a new yet general memristive neural network model, namely, discrete-time stochastic delayed MNN, is proposed in order to reflect the engineering practice; (2) the leakage and probabilistic delays are, for the first time, handled simultaneously in DSMMNs by choosing an appropriate Lyapunov–Krasovskii functional; and (3) delay-dependent conditions for exponential stability of the addressed DSMNN are obtained by employing up-to-date dynamics analysis techniques*.

Notation. The notation used here is fairly standard except where otherwise stated. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n\times m$ real matrices. I denotes the identity matrix of compatible dimension. The notation $X \geq Y$ (respectively, $X > Y$), where X and Y are system symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive definite). For a matrix A, A^T represents the transpose of A, $\lambda_{\min}\{A\}$ ($\lambda_{\max}\{A\}$) denote the smallest (largest) eigenvalue of A. diag$\{\cdots\}$ stands for a block-diagonal matrix. $\mathbb{E}\{x\}$ stands for the expectation of the stochastic variable x. $\|x\|$ describes the Euclidean norm of a vector x. Matrices, if they are not explicitly specified, are assumed to have compatible dimensions. Sometimes, the arguments of a function will be omitted in the analysis when no confusion can arise.

2. Problem formulation

Consider the following DSMNN with constant leakage delay and probabilistic time-varying delays:

$$\begin{aligned} x(k+1) = {} & D(x(k))x(k-\ell) + A(x(k))f(x(k)) \\ & + B(x(k))g(x(k-\tau(k))) \\ & + \sigma(k, x(k), x(k-\tau(k)))w(k) \end{aligned} \tag{1}$$

where $x(k) = \begin{bmatrix} x_1(k) & x_2(k) & \cdots & x_n(k) \end{bmatrix}^T$ is the neuron state vector; $D(x(k)) = \text{diag}\{d_1(x_1(k)), d_2(x_2(k)), \ldots, d_n(x_n(k))\}$ is the self-feedback matrix with entries $d_i(x_i(k)) > 0$; $A(x(k)) = \left(a_{ij}(x_i(k))\right)_{n\times n}$