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Global exponential synchronization of inertial memristive neural networks with time-varying delay via nonlinear controller[☆]

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ABSTRACT

The paper is concerned with the synchronization problem of inertial memristive neural networks with time-varying delay. First, by choosing a proper variable substitution, inertial memristive neural networks described by second-order differential equations can be transformed into first-order differential equations. Then, a novel controller with a linear diffusive term and discontinuous sign term is designed. By using the controller, the sufficient conditions for assuring the global exponential synchronization of the derive and response neural networks are derived based on Lyapunov stability theory and some inequality techniques. Finally, several numerical simulations are provided to substantiate the effectiveness of the theoretical results.

1. Introduction

According to the relationship between the charge and magnetic flux in the circuit, Chua (1971) predicted that there exists the fourth fundamental circuit element which is called memristor. In 2008, the first real memristor device was confirmed by the researchers of Hewlett–Packard (Strukov, Snider, & Stewart, 2008). After that, more and more researches began to pay attention to the memristor. Due to its special properties, such as nanoscale, low energy dissipation and memory ability, memristor can be applied in pattern recognition (Sharifi & Banadaki, 2010), image processing (Chen, Zeng, & Jiang, 2014; Kim, Sah, Yang, & Roska, 2012), and optimization problems (Wen & Zeng, 2012). In particular, it can simulate synapse among the neurons better than resister in the circuit implementation of neural networks (Wang, Li, Peng, Xiao, & Yang, 2014). By replacing resister with memristor in the circuit implement of neural network, we can obtain memristive neural networks which are very suitable to simulate human brain (Jo et al., 2010).

As we all know, the synchronization is one of the most important dynamical behaviors of neural networks (He, Qian, Han, & Cao, 2011). Due to its wide applications, including secure communications (Lakshmanan, Prakash, & Lim, 2018), associative memory (Tan & Ali, 2011), and biological networks (He & Cao, 2009), the synchronization of neural networks has attracted great attention of researchers. Recently, many related results on the synchronization problem of MNNs have been published (Abdurahman & Jiang, 2016; Guo, Wang, & Yan, 2015; Zheng & Xian, 2016). In Zheng and Xian (2016), a linear delay-dependent state feedback controller was designed to ensure that chaotic MNNs can be globally asymptotically synchronized by some inequality techniques. In Guo et al. (2015), the authors investigated the global exponential synchronization of memristive recurrent neural networks (MRNNs) with time delays by using four different control strategies. The authors in Abdurahman and Jiang (2016) studied the exponential synchronization of delayed MNNs with discontinuous activation functions by using some new techniques and differential inclusion theory.

The inertial term in nonlinear system is taken as a critical tool to generate bifurcation and chaos (He, Li, & Shu, 2012). Therefore, inertial neural networks may exhibit more complicated dynamical behaviors, such as chaos, which is important in secure communication. The concept of inertial neural networks was put forward by Babcock and Westervelt at 1987 (Babcock & Westervelt, 1987). Many results on synchronization of inertial neural networks have been reported (Cao & Wan, 2014; Dharani, Rakkiyappan, & Park, 2017; Hu, Cao, & Alofi, 2015). However, as far as we know, there are fewer results on synchronization of inertial memristive neural networks (Rakkiyappan, Kumari, Chandrasekar, & Krishnasamy, 2016; Rakkiyappan, Premalatha, Chandrasekar, & Cao, 2016). In

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Rakkiyappan, Kumari et al. (2016), by employing the second order differential inclusion theory and matrix measure method, the periodicity and synchronization of inertial memristive neural networks with supremums and time delays was investigated. The authors in Rakkiyappan, Premalatha et al. (2016) studied the stability and pinning synchronization of inertial memristive neural networks with time delay by using differential inclusion theory and matrix measure strategy. In these two papers, the activation functions f_i, $i = 1, 2, \ldots, n$ are assumed to satisfy $f_i(\pm T) = 0$, where T is the switching threshold. This assumption is unreasonable since some classical activation functions, such as $f(x) = \frac{|x+1|-|x-1|}{2}$ and $\tanh(x)$, do not satisfy it. Then, a natural question arises: would this assumption be removed?

Motivated by the above discussions, the purpose of this paper is to investigate the problem of global exponential synchronization of inertial MNNs with time-varying delay. Inertial memristive neural network can be converted to first-order differential equations by introducing a variable transformation. Different from the existed results (Rakkiyappan, Kumari et al., 2016; Rakkiyappan, Premalatha et al., 2016), in this paper, a novel and simple nonlinear controller is put forward to ensure that the global exponential synchronization between drive system and response system is realized. Based on Lyapunov stability theory and some inequality techniques, several synchronous criteria are derived without assuming that the activation functions are equal to zero at the switching threshold.

The paper is organized as follows. In Section 2, some preliminaries are provided. The main results are derived in Section 3. In Section 4, some numerical simulations are presented to substantiate the theoretical results. Finally, conclusions are drawn in Section 5.

2. Preliminaries

We consider the following inertial memristive neural network with time-varying delay:

$$\begin{aligned}\frac{d^2u_i(t)}{dt^2} =& - a_i\frac{du_i(t)}{dt} - b_iu_i(t)\\ &+ \sum_{j=1}^{n} c_{ij}(f_j(u_j(t)) - u_i(t))f_j(u_j(t))\\ &+ \sum_{j=1}^{n} d_{ij}(f_j(u_j(t-\tau(t))) - u_i(t))f_j(u_j(t-\tau(t)))\\ &+ I_i, \quad \text{for } i = 1, 2, \ldots, n, \end{aligned} \tag{1}$$

where $u_i(t)$ is the state of the ith neuron at time t; the second-order derivative of $u_i(t)$ is called an inertial term; $a_i > 0$ and $b_i > 0$ are constants; $f_i(\cdot)$ denotes the activation function of the ith neurons; $\tau(t)$ is the time-varying transmission delay and satisfies $0 < \tau_0 \le \tau(t) \le \tau$; I_i denotes the external input on the ith neuron; $c_{ij}(f_j(u_j(t)) - u_i(t))$ and $d_{ij}(f_j(u_j(t-\tau(t))) - u_i(t))$ represent the memristive feedback connection weight and delayed feedback connection weight, respectively.

For convenience, we denote $c_{ij}(u_i) = c_{ij}(f_j(u_j(t)) - u_i(t))$, $d_{ij}(u_i) = d_{ij}(f_j(u_j(t-\tau(t))) - u_i(t))$, and $f_{ij}(t) = f_j(u_j(t)) - u_i(t)$, $f_{ij}(t-\tau(t)) = f_j(u_j(t-\tau(t))) - u_i(t)$. As we know, the connection weights are realized by memristors in the circuit implementation. According to pinched hysteretic feature of memristor, we provide a mathematical model of the connection weights as follows:

$$c_{ij}(u_i) = \begin{cases} c'_{ij}, & f_{ij}^{-}(t) > 0,\\ c''_{ij}, & f_{ij}^{-}(t) < 0,\\ \lim\limits_{s\to t^-} c_{ij}(f_{ij}(s)), & f_{ij}^{-}(t) = 0 \end{cases}$$

and

$$d_{ij}(u_i) = \begin{cases} d'_{ij}, & f_{ij}^{-}(t-\tau(t)) > 0,\\ d''_{ij}, & f_{ij}^{-}(t-\tau(t)) < 0,\\ \lim\limits_{s\to t^-} d_{ij}(f_{ij}(s-\tau(s))), & f_{ij}^{-}(t-\tau(t)) = 0, \end{cases}$$

for $i, j = 1, 2, \ldots, n$, where $f_{ij}^{-}(\cdot)$ denotes the left derivation of $f_{ij}(\cdot)$. Moreover, denote $\hat{c}_{ij} = \max\{c'_{ij}, c''_{ij}\}$, $\check{c}_{ij} = \min\{c'_{ij}, c''_{ij}\}$, $\hat{d}_{ij} = \max\{d'_{ij}, d''_{ij}\}$, $\check{d}_{ij} = \min\{d'_{ij}, d''_{ij}\}$, $\bar{c}_{ij} = \max\{|c'_{ij}|, |c''_{ij}|\}$ and $\bar{d}_{ij} = \max\{|d'_{ij}|, |d''_{ij}|\}$.

The initial value of (1) is given as

$$\begin{cases} u_i(s) = \phi_i(s),\\ \dfrac{du_i(s)}{dt} = \psi_i(s), \qquad -\tau \le s \le 0, \end{cases} \tag{2}$$

where $\phi_i(s), \psi_i(s) \in C([-\tau, 0], \mathbb{R})$ which denotes the set of all continuous functions mapping the interval $[-\tau, 0]$ into $\mathbb{R}$.

Remark 1. According to the definition of connection weights, $c_{ij}(u_i)$ and $d_{ij}(u_i)$ in system (1) vary dependently in the state. Therefore, system (1) can be treated as a second-order state-dependent switching system. When $c'_{ij} = c''_{ij}$, $d'_{ij} = d''_{ij}$, system (1) will become a traditional inertial neural network, see He et al. (2012) Li, Chen, and Liao (2004) and Liu, Liao, Liu, Zhou, and Guo (2009).

Since the classical definition of solution is invalid due to the discontinuity of memristive connection weights, we need to introduce the definition of solution in sense of Fillipov for system (1).

A function $u(t) = (u_1(t), u_2(t), \ldots, u_n(t))^T : [-\tau, T) \to \mathbb{R}^n$, $T \in (0, +\infty]$ is a solution (in the sense of Filippov) of system (1) with initial condition (2), if $u(t)$ is continuous on $[-\tau, T)$ and absolutely continuous on any compact subinterval of $[0, T)$, and for almost all (a.a.) $t \in [0, T)$ satisfies

$$\begin{aligned}\frac{d^2u_i(t)}{dt^2} \in& - a_i\frac{du_i(t)}{dt} - b_iu_i(t) + \sum_{j=1}^{n} \overline{\text{co}}[c_{ij}(u_i)]f_j(u_j(t))\\ &+ \sum_{j=1}^{n} \overline{\text{co}}[d_{ij}(u_i)]f_j(u_j(t-\tau(t))) + I_i, \end{aligned} \tag{3}$$

for $i = 1, 2, \ldots, n$, where $\overline{\text{co}}[c_{ij}(u_i)] = [\check{c}_{ij}, \hat{c}_{ij}]$ and $\overline{\text{co}}[d_{ij}(u_i)] = [\check{d}_{ij}, \hat{d}_{ij}]$.

Next, we introduce a variable transformation as follows:

$$y_i(t) = \xi_i\frac{du_i(t)}{dt} + u_i(t),$$

where $\xi_i > 0$. Without loss of generality, let $\xi_i = 1$, then, for a.a. $t \in [0, T)$, we have

$$\begin{aligned}\frac{dy_i(t)}{dt} \in& - [b_i + 1 - a_i]u_i(t) - (a_i - 1)y_i(t)\\ &+ \sum_{j=1}^{n} \overline{\text{co}}[c_{ij}(u_i)]f_j(u_j(t))\\ &+ \sum_{j=1}^{n} \overline{\text{co}}[d_{ij}(u_i)]f_j(u_j(t-\tau(t))) + I_i. \end{aligned}$$

Hence, system (3) can be rewritten as

$$\begin{cases} \dfrac{du_i(t)}{dt} = -u_i(t) + y_i(t),\\ \dfrac{dy_i(t)}{dt} \in -\alpha_iu_i(t) - \beta_iy_i(t) + \displaystyle\sum_{j=1}^{n} \overline{\text{co}}[c_{ij}(u_i)]f_j(u_j(t))\\ \qquad + \displaystyle\sum_{j=1}^{n} \overline{\text{co}}[d_{ij}(u_i)]f_j(u_j(t-\tau(t))) + I_i, \end{cases} \tag{4}$$