Neural Networks 101 (2018) 25–32

Contents lists available at ScienceDirect

Neural Networks

journal homepage: www.elsevier.com/locate/neunet

Unified synchronization criteria in an array of coupled neural networks with hybrid impulses

Nan Wang [a], Xuechen Li [a], Jianquan Lu [b,*], Fuad E. Alsaadi [c]

[a] School of Mathematics and Statistics, Xuchang University, Xuchang 461000, China
[b] School of Mathematics, Southeast University, Nanjing 210096, China
[c] Department of Electrical and Computer Engineering, Faculty of Engineering, King Abdulaziz University, Jeddah 21589, Saudi Arabia

ARTICLE INFO

Article history:
Received 21 September 2017
Received in revised form 24 December 2017
Accepted 30 January 2018
Available online 7 February 2018

Keywords:
Coupled neural networks
Synchronization
Average impulsive interval
Average impulsive gain
Hybrid impulses

ABSTRACT

This paper investigates the problem of globally exponential synchronization of coupled neural networks with hybrid impulses. Two new concepts on average impulsive interval and average impulsive gain are proposed to deal with the difficulties coming from hybrid impulses. By employing the Lyapunov method combined with some mathematical analysis, some efficient unified criteria are obtained to guarantee the globally exponential synchronization of impulsive networks. Our method and criteria are proved to be effective for impulsively coupled neural networks simultaneously with synchronizing impulses and desynchronizing impulses, and we do not need to discuss these two kinds of impulses separately. Moreover, by using our average impulsive interval method, we can obtain an interesting and valuable result for the case of average impulsive interval $T_a = \infty$. For some sparse impulsive sequences with $T_a = \infty$, the impulses can happen for infinite number of times, but they do not have essential influence on the synchronization property of networks. Finally, numerical examples including scale-free networks are exploited to illustrate our theoretical results.

1. Introduction

A complex dynamical network is a large set of coupled nodes, in which each node represents an individual element in the network and the edges represent the relations among the nodes. As an interesting behavior of complex networks, synchronization of complex networks has been extensively investigated in Arenas, Díaz-Guilera, Kurths, Moreno, and Zhou (2008), He, Qian, and Cao (2017), Li, Ho, Cao, and Lu (2016), Liang, Dai, Shen, Wang, Wang, and Chen (2018), Lu and Ho (2010), Lu, Ho, and Wu (2009), Ma, Wang, and Lu (2012), Pecora and Carroll (1998), Wu (2007), and Zhong, Lu, Huang, and Ho (2017) over the last decade because of its potential applications, such as pattern storage and retrieval (Hoppensteadt & Izhikevich, 2000), and parallel image processing (Krinsky, Biktashev, & Efimov, 1991). In He, Qian, Lam, Chen, Han, and Kuergen (2015), He et al. investigated the quasi-synchronization of heterogeneous dynamical networks, and obtained some interesting results. Using different analytical techniques, many important criteria have been obtained about synchronization with some special features, such as fractional-order dynamics (Huang, Fan, Jia, Wang, & Li, 2017), observer-design (Zhang, Shao, Wang, & Shen, 2012), stochastic phenomena (Li, 2017; Lu, Ho, & Wang, 2009; Lu, Kurths, Cao, Mahdavi, & Huang, 2012; Xu, Lu, Peng, Xie, & Xue, 2017), time delays (Li & Wu, 2016; Lu, Wang, Cao, Ho, & Kurths, 2012; Song, Yan, Zhao, & Liu, 2016b; Yang, Ho, Lu, & Song, 2015), switching behavior (Cheng, Chen, Qiu, Lu, & Cao, in press), noise (Lu & Ho, 2011), multi-layer feature (He, Chen, Han, Du, Cao, & Qian, 2017) and impulsive effects (He, Qian et al., 2017; Li & Song, 2017; Song, Yan, Zhao, & Liu, 2016a; Wang, Li, Huang, & Chen, 2014). In He, Chen, Han, and Qian (2017), considering network-induced delays, He et al. designed distributed impulsive control to solve the problem of leader-following consensus. In Wang, Yu, Li, Wang, Huang, and Huang (2015), Wang et al. investigated the stability problem of delayed neural networks with impulsive time window. Impulsive effects mean that the states of nodes are often subject to instantaneous perturbations and experience abrupt change at certain instants, which may be caused by switching phenomenon, frequency change or other sudden noises (Wang, Lu, Lou, Ding, Alsaadi, & Hayat, in press; Yang, 2001; Zhang, Meng, Feng, & Zhang, 2017). Impulsive effects widely exist in biological networks. Such systems can be well described by impulsive differential systems which have been used successfully to model many practical problems in the fields of natural sciences and technology (Yang, 2001; Yang, Cao, & Qiu, 2015; Zhang, Guan, & Feng, 2008). Neural networks have been widely studied (Wang, Wang, Li, & Huang, in press; Xu, Wang, Yao, Lu, & Su, 2017). Neural networks that use impulse trains for connections between neurons reflect the

* Corresponding author.
E-mail address: jqluma@seu.edu.cn (J. Lu).

https://doi.org/10.1016/j.neunet.2018.01.017
0893-6080/

structure found in biological nervous systems. Real nerve cells transmit electrical impulses along their axons (Dayhoff, 1988). Since impulses can greatly affect the dynamical behaviors of nervous networks, it is necessary to investigate impulsive effects on the synchronization of coupled neural networks.

Generally, with regard to synchronization, there are two kinds of impulses in complex dynamical networks (Lu, Ho, & Cao, 2010). An impulsive sequence is said to be desynchronizing if the impulsive effect can suppress the synchronization of complex dynamical networks. An impulsive sequence is said to be synchronizing if corresponding impulsive effect can enhance the synchronization of complex dynamical networks. In previous literature (Liu, Liu, Chen, & Wang, 2005), almost all of the results are devoted to studying these two kinds of impulses separately by using the lower bound or the upper bound of the impulsive intervals, and hence the obtained results for synchronizing impulses cannot be applied to study dynamical networks with desynchronizing impulses. In Lu et al. (2010), Lu et al. have given a unified synchronization criterion which is simultaneously valid for these two kinds of impulses.

Characterizing the frequency of impulses with lower bound or upper bound of the impulsive intervals would lead to very conservative results. Hence, Lu et al. (2010) has sought out a more accurate description about impulses' occurrence with the novel concept named average impulsive interval. By means of average impulsive interval, some good criteria have been derived for the synchronization of networks (Lu, Ding, Lou, & Cao, 2015). However, to the best of our knowledge, most of previous literature are devoted to investigating the synchronization of impulsive dynamical networks either with impulse gain $|\mu_k| > 1$ (desynchronizing impulses) or with $|\mu_k| < 1$ (synchronizing impulses), separately. In Hespanha, Liberzon, and Teel (2005), stability of impulsive systems is studied, and desynchronizing impulses and synchronizing impulses are separately considered. However, under many circumstances, this simplification does not match the peculiarities of real networks. It means that there exist both $|\mu_k| > 1$ and $|\mu_k| < 1$ in the same impulsive sequence, so it is necessary to study both kinds of impulses simultaneously. In Wong, Zhang, Tang, and Wu (2013) and many other references, the authors obtained some interesting results about synchronization of networks with delay coupling and mixed impulses. Inspired by the above-mentioned discussions, a new concept of hybrid impulse will be introduced in this paper to describe more general impulsive sequences, which can simultaneously permit $|\mu_k| > 1$ and $|\mu_k| < 1$. Moreover, to deal with the difficulties from hybrid impulses, the average impulse gain will be proposed and well utilized to study the synchronization of networks with hybrid impulses in this paper.

In addition, average impulsive interval proposed in Lu et al. (2010) does not have an intuitive feeling. Considering this issue, inspired by Lu et al. (2010), we introduce a new average impulsive interval in the form of limit. On one hand, the new average impulsive interval here is weaker than the definition of average impulsive interval in Lu et al. (2010), and further our result is more general than that of Lu et al. (2010). On the other hand, the situation $T_a = \infty$ can emerge when impulses occur infinitely but sparsely. Considering this situation, from the concept on average impulsive interval, we proposed $T_a = \infty$ to describe that impulses occur infinitely but sparsely, which was not considered in Lu et al. (2010). More importantly, an interesting and valuable result concerning globally exponential synchronization of hybrid impulsive dynamical networks can be obtained when average impulsive interval $T_a = \infty$. Our result reveals that some kinds of impulsive sequences contain infinite number of impulses, but cannot essentially influence the synchronization property of networks.

Hence, the synchronization problem for impulsive neural networks has not been completely investigated, and it is still open and remains challenging. In this paper, we will use a novel method named average impulsive gain to solve the synchronization problem of networks simultaneously with two kinds of impulses. The remainder of this paper is organized as follows. Section 2 describes the model of the dynamical networks with hybrid impulses and gives some preliminaries. The synchronization criteria are presented in Section 3. Section 4 provides two illustrative examples including scale-free network to illustrate our theoretical results. Finally, conclusions are made in Section 5.

Notations: The standard notations will be used throughout this paper. I_n denotes the $n \times n$ identity matrix. $\lambda_{\max}(\cdot)$ represents the largest eigenvalue of the corresponding matrix. $\mathbb{N} = \{1, 2, 3, \ldots\}$. $\mathbb{R}^n$ denotes the n dimensional Euclidean space. $\mathbb{R}^{n\times n}$ represents the $n \times n$ real matrices. The notation "T" denotes the transpose of a matrix or a vector. $\|x\|$ indicates the 2-norm of a vector x, i.e., $\|x\| = (\sum_{i=1}^{n} x_i^2)^{\frac{1}{2}}$. $[t]$ represents the maximum integer of no more than t. $\otimes$ denotes the Kronecker-product. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

2. Model description and some preliminaries

In this section, some preliminaries including model formulation, definitions, and lemmas are described.

Consider a complex dynamical network consisting of N linearly coupled identical neural networks. Each node is an n-dimensional neural network. A single neural network can be described as follows:

$$\dot{s}(t) = Cs(t) + Bf(s(t)), \tag{1}$$

where $s(t) = [s_1(t), s_2(t), \ldots, s_n(t)]^T$ is the state vector at time t; $C \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times n}$, and $f(s(t)) = [f_1(s(t)), f_2(s(t)), \ldots, f_n(s(t))]^T$.

When N neural networks are coupled together, and taking impulsive time effects into account and let $a_{ii} = -\sum_{j=1, j\neq i}^{N} a_{ij}$, $i = 1, 2, \ldots, N$, the impulsive dynamical network can be inferred in the following form:

$$\begin{cases} \dot{x}_i(t) = Cx_i(t) + Bf(x_i(t)) + c\displaystyle\sum_{j=1}^{N} a_{ij}\Gamma x_j(t), \\ \quad t \geq 0,\ t \neq t_k,\ k \in \mathbb{N}, \\ x_j(t_k^+) - x_i(t_k^+) = \mu_k(x_j(t_k^-) - x_i(t_k^-)), \\ \quad \text{for } (i, j) \text{ satisfying } a_{ij} > 0, \end{cases} \tag{2}$$

where $x_i(t) = [x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t)]^T$ is the state vector of the ith node at time t; Γ is the inner-coupling positive definite matrix; $c > 0$ is the coupling strength; and the a_{ij} is defined as follows: if there is a connection from neuron j to neuron i ($j \neq i$), then $a_{ij} > 0$; otherwise, $a_{ij} = 0$.

The fixed moments of time t_k satisfy $t_{k-1} < t_k$ and $\lim t_k \to +\infty$ as $k \to +\infty$. $\zeta = \{t_1, t_2, t_3, \ldots\}$ is an impulsive sequence and μ_k is the strength of impulsive signal. Here we permit $|\mu_k| > 1$ and $|\mu_k| < 1$ simultaneously; $A = (a_{ij})_{N\times N}$ is the Laplacian matrix representing the topology of the corresponding network (Chung, 1997).

We need the following definitions, assumptions and lemma for the derivation of the synchronization criteria.

Definition 1. The neural coupled network (2) is said to be globally exponentially synchronized if there exist $\eta > 0$, $T > 0$ and $M_0 > 0$, such that for any initial values,

$$\|x_i(t) - x_j(t)\| \leq M_0 e^{-\eta t}$$

holds for all $t > T$, and for any $i, j = 1, 2, \ldots, N$.

Definition 2 (*Average Impulsive Gain*)**.** The average impulsive gain is defined as follow:

$$\mu = \lim_{t\to+\infty} \frac{|\mu_1| + |\mu_2| + \cdots + |\mu_{N_\zeta(t,t_0)}|}{N_\zeta(t, t_0)} > 0, \tag{3}$$