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$O(t^{-\alpha})$-synchronization and Mittag-Leffler synchronization for the fractional-order memristive neural networks with delays and discontinuous neuron activations

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ABSTRACT

This paper investigates $O(t^{-\alpha})$-synchronization and adaptive Mittag-Leffler synchronization for the fractional-order memristive neural networks with delays and discontinuous neuron activations. Firstly, based on the framework of Filippov solution and differential inclusion theory, using a Razumikhin-type method, some sufficient conditions ensuring the global $O(t^{-\alpha})$-synchronization of considered networks are established via a linear-type discontinuous control. Next, a new fractional differential inequality is established and two new discontinuous adaptive controller is designed to achieve Mittag-Leffler synchronization between the drive system and the response systems using this inequality. Finally, two numerical simulations are given to show the effectiveness of the theoretical results. Our approach and theoretical results have a leading significance in the design of synchronized fractional-order memristive neural networks circuits involving discontinuous activations and time-varying delays.

1. Introduction

The fractional calculus has become a great research topic in recent years due to its many applications in the field of physics and engineering (Kilbas, Srivastava, & Trujillo, 2006; Podlubny, 1999). In fact, many of the real world objects are generally identified and described by the fractional-order model. This model is more accurate than the integer-order model. The main advantage of fractional-order model in comparison with integer-order model is that a fractional derivative provides an excellent tool in the description of memory and hereditary properties of various processes. In addition, the fractional-order model has more degrees of freedom and unlimited memory (infinite memory). Based on these features, some researchers have introduced fractional calculus in neural network models to form a fractional-order neural network model. Therefore, it is needed to study the dynamics of fractional-order neural networks. For the past few years, the analysis of fractional-order neural networks has become an increasing interest and growing area of research, and the dynamical behaviors of fractional-order neural networks, such as synchronization, stability, and state estimation have been discussed in Refs. Chen and Chen (2015a, 2015b, 2016), Qi, Li, and Huang (2014), Rakkiyappan, Velmurugan, and Cao (2015), Rakkiyappan, Velmurugan, Rihan, and Lakshmanan (2016), Yan, Cao, and Liang (2016) and Yang and DWC (2016) and references therein.

Memristor is a contraction of memory resistor, which is a new nonlinear electric circuit element, that describes the relationship between electric charge and magnetic flux. The memristors were first introduced theoretically by Chua (Chua, 1971) and it has been realized practically by the research team of HP Lab in 2008 (Strukov, Snider, Stewart, & Williams, 2008). Memristor is a two-terminal element with variable resistance and its value is not unique, which depends on the magnitude and polarity of the voltage applied to it and the length of the time that the voltage has been applied. When the voltage is turned off, the memristor remembers its most recent value until next time it is turned on. Therefore, memristors have been used for nonvolatile memory storage. Based on the memristors, a new type of neural network model, called the memristor-based neural networks, has been introduced in the literature and dynamical behaviors have been investigated (see Abdurahman and Jiang (2016), Abdurahman, Jiang, and Rahman (2015), Chandrasekar, Rakkiyappan, Cao, and Lakshmanan (2014), Chen, Zeng, and Jiang (2014a, b, c), Hu and Wang (2010) and

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Wang, Li, Peng, Xiao, and Yang (2014) and references therein). In addition, the analysis of memristor-based neural networks is necessary on account of its potential applications in next generation computer and powerful brain-like neural computer (Chua, 1971; Wang, Chen, Xi, Li, & Dimitrov, 2009).

As is well known, the chaos synchronization has received great attention in the investigation of neural networks since its successful applications in a variety of fields. Therefore, the synchronization phenomenon of neural networks is an important issue that is explored by many researchers (see, e.g., Refs. Abdurahman and Jiang (2016), Abdurahman et al. (2015), Chandrasekar et al. (2014), Li and Cao (2015), Mathiyalagan, Park, and Sakthivel (2015), Shen, Wu, and Park (2015) and Stamova (2014)). Also, the synchronization of fractional-order dynamical systems becomes a stimulating and inspiring problem due to its potential applications in ranging from computer science to biology, from physics to engineering, even from economics to brain science, secure communication, and control processing (see Refs. Bao, Park, and Cao (2015, 2016), Velmurugan and Rakkiyappan (2016) and Velmurugan, Rakkiyappan, and Cao (2016)).

Time delays, especially time-varying delays, are unavoidably encountered in the signal transmission among the neurons, which will affect the stability of neural networks and may lead to some complex dynamic behaviors (see Ahn, Shi, and Wu (2015), Saravanakumar, Ali, Ahn, Karimi, and Shi (2017) and references therein). In Chen and Chen (2015a) we have studied global $O(t^{-\alpha})$ stability and global asymptotical periodicity for a non-autonomous fractional-order neural networks with time-varying delays by a Razumikhin-type method (see Chen and Chen (2015b)). In Bao et al. (2015) Bao et al. discussed the adaptive synchronization of fractional-order memristor-based neural networks with time delay by combining the adaptive control, linear delay feedback control, and a fractional-order inequality. The results on exponential synchronization of memristor-based neural networks with delay and discontinuous neuron activations are established via two types of discontinuous controls: linear feedback control and adaptive control in Abdurahman and Jiang (2016). However, to the best of our knowledge, there are very few or even no results on the $O(t^{-\alpha})$-synchronization and Mittag-Leffler synchronization of fractional-order memristive neural networks with delay and discontinuous neuron activations. Motivated by the previous works and background, the main purpose of this paper is to fill this gap. The present paper at least have four highlights as follows: (1) Two new types of synchronization, $O(t^{-\alpha})$-synchronization and Mittag-Leffler synchronization, are proposed, which can better describe synchronization feature of fractional-order systems. (2) Some sufficient conditions ensuring the global $O(t^{-\alpha})$-synchronization of considered networks are established via a linear-type discontinuous control. (3) A new fractional differential inequality is established and two new discontinuous adaptive controllers are designed to achieve Mittag-Leffler synchronization between the drive system and the response systems using this inequality. (4) The works are new that fill some gap of the existing works. (5) Our results generalize and improve those of existing literature.

The rest of the paper is organized as follows. In Section 2, the drive–response systems are introduced. In addition, some assumptions and definitions together with some useful lemmas needed in this paper are presented. In Section 3, we devote to investigating the $O(t^{-\alpha})$-synchronization between the drive system and the response systems by designing a linear-type discontinuous controller. In Section 4, a new discontinuous feedback controller is designed to achieve Mittag-Leffler synchronization between the drive system and the response systems. In Section 5, two numerical examples and their simulations are given to illustrate the effectiveness of the obtained results. Finally, some general conclusions are drawn in Section 6.

2. Preliminaries

In order to describe our model, we will recall some definitions of fractional calculation.

The fractional integral with order α for a function $f(t)$ is defined as

$$ {}_{t_0}^{RL}D_t^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_{t_0}^{t} (t-s)^{\alpha-1} f(s)ds $$

where $t \geq t_0$ and $\alpha > 0$, $\Gamma(\cdot)$ is the gamma function, that is

$$ \Gamma(\alpha) = \int_0^{\infty} t^{\alpha-1} e^{-t} dt. $$

The Riemann–Liouville derivative of fractional with order α of function $f(t)$ is given as

$$ \begin{aligned} {}_{t_0}^{RL}D_t^{\alpha} f(t) &= \frac{d^n}{dt^n} D_{t_0,t}^{-(n-\alpha)} f(t) \\ &= \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_{t_0}^{t} \frac{f(s)}{(t-s)^{\alpha-n+1}} ds. \end{aligned} $$

The Caputo's fractional derivative with order α for a function $f \in C^{n+1}([t_0, +\infty), R)$ is defined by

$$ D_{t_0}^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_{t_0}^{t} \frac{f^{(n)}(s)}{(t-s)^{\alpha-n+1}} ds $$

where $t > t_0$ and n is a positive integer such that $n-1 < \alpha < n$.

In this paper, we consider a class of delayed fractional-order memristive neural network (DFMNN) with discontinuous activation functions described by the following equation:

$$ \begin{aligned} D_{t_0}^{\alpha} x_i(t) &= -c_i(x_i(t))x_i(t) + \sum_{j=1}^{n} a_{ij}(x_j(t)) f_j(x_j(t)) \\ &\quad + \sum_{j=1}^{n} b_{ij}(x_j(t-\tau_{ij}(t))) g_j(x_j(t-\tau_{ij}(t))) + I_i, \end{aligned} \tag{2.1} $$

where $i = 1, \ldots, n$, $t \geq 0$ $0 < \alpha < 1$, n corresponds to the number of units in a neural network, $x_i(t)$ denotes the state variable associated with the ith neuron, I_i denotes the external input, which is a constant, $\tau_{ij}(t)$ is a nonnegative function representing the finite speed of the axonal signal transmission in the time t, $f_i(\cdot)$ and $g_i(\cdot)$ are all the nonlinear activation function, that can be discontinuous; $c_i(x_i)$, $a_{ij}(x_j)$ and $b_{ij}(x_j)$ $(i, j = 1, \ldots, n)$ are, respectively, the memristor-based connection weights and those associated with time delays, that are given by

$$ a_{ij}(\xi) = \begin{cases} \hat{a}_{ij}, & |\xi| > T_j^a, \\ \text{unsureness}, & |\xi| = T_j^a, \\ \check{a}_{ij}, & |\xi| < T_j^a, \end{cases} \tag{2.2} $$

$$ b_{ij}(\xi) = \begin{cases} \hat{b}_{ij}, & |\xi| > T_j^b, \\ \text{unsureness}, & |\xi| = T_j^b, \\ \check{b}_{ij}, & |\xi| < T_j^b, \end{cases} \tag{2.3} $$

and

$$ c_i(\xi) = \begin{cases} \hat{c}_i, & |\xi| > T_j^c, \\ \text{unsureness}, & |\xi| = T_j^c, \\ \check{c}_i, & |\xi| < T_j^c, \end{cases} \tag{2.4} $$