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Multiple types of synchronization analysis for discontinuous Cohen–Grossberg neural networks with time-varying delays ☆

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A B S T R A C T

This paper is devoted to the exponential synchronization, finite time synchronization, and fixed-time synchronization of Cohen–Grossberg neural networks (CGNNs) with discontinuous activations and time-varying delays. Discontinuous feedback controller and Novel adaptive feedback controller are designed to realize global exponential synchronization, finite time synchronization and fixed-time synchronization by adjusting the values of the parameters ω in the controller. Furthermore, the settling time of the fixed-time synchronization derived in this paper is less conservative and more accurate. Finally, some numerical examples are provided to show the effectiveness and flexibility of the results derived in this paper.

1. Introduction

The synchronization of systems means that the dynamical behaviors of coupled systems achieve the same time spatial state, which plays an important role in various disciplines such as information processing, biology systems and linguistic networks (Bao & Cao, 2014; Guo, Yang, & Wang, 2014; Wen, Zeng, Huang, & Chen, 2013; Yang, Guo, & Wang, 2016). Recently, the synchronization problem of CGNNs has attracted considerable attention for its potential applications in signal processing and secure communication. A variety of results for exponential synchronization, lag synchronization, complete synchronization and anti-synchronization have been derived under varied control methods, such as adaptive control (Ding & Fu, 2008; Yang & Cao, 2013b; Zhou, Chen, & Xiang, 2006), feedback control (Cui & Lou, 2009; Hu, Yu, & Jiang, 2014), intermittent control (Feng, Yang, & Zhao, 2016; Yu, Hu, & Jiang, 2011; Zhang & Shen, 2014). However, most of existing results on synchronization of neural networks including the articles mentioned above are actually asymptotic results. Recently, there are two categories of concepts of synchronization over finite time control.

One is finite time synchronization, which means that the drive–response systems are synchronized within a finite time interval for any initial values. Finite-time synchronization has many applications. In practical engineering situation, it is desired to synchronize a dynamical system achieved in finite time. Such as in secure communication, fast observation and estimation, fast adaptation algorithms or fast consensus protocols etc. Thus, finite time control as a kind of time optimal control has drawn an increasing attention. In Jiang, Wang, Mei, and Shen (2014), the authors designed finite-time synchronization controllers for a class of memristor-based recurrent neural networks and some useful sufficient conditions were derived by using differential inclusions theory and Lyapunov functional method. Subsequently, Cao et al. in Velmurugan, Rakkiyappan, and Cao (2016) investigated finite time synchronization of fractional-order memristor-based neural networks with time delays through Laplace transform, generalized Gronwall's inequality, and feedback control schemes. In addition, Shen and Cao (2011) discussed finite time synchronization of coupled neural networks via discontinuous controllers. Besides, there are many articles on finite time synchronization (Aghababa & Aghababa, 2012; Aghababa, Khanmohammadi, & Alizadeh, 2011; Chen, Li, Yang, Yang, & Li, 2017; Gao, Zhu, Alsaedi, Alsaadi, & Hayat, 2016; Liu, Cao, Yu, & Song, 2016; Liu, Su, & Chen, 2016).

The other is fixed-time synchronization, which not only claims that the investigated drive–response system achieve synchronization within a finite time segment, but also needs a uniform upper bound for such time intervals of all initial synchronization errors. In fact, fixed-time synchronization is proposed based on the finite-time stability by demanding the boundness of the settling time function. In Polyakov (2012), fixed-time stable is firstly introduced by Polyakov based on nonlinear feedback control. Further investigations of fixed-time stability and synchronization problems have been presented in Cao and Li (2017), Khanzadeh and Pourgholi

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(2017), Lu, Liu, and Chen (2016), Ni, Liu, Liu, Hu, and Shen (2016) and Parsegov, Polyakov, and Shcherbakov (2013). By designing a new controller and based on Lyapunov functions and analytical techniques, the fixed-time synchronization of delayed memristor-based recurrent neural networks is solved in Cao and Li (2017). In Wang, Zeng, Hu, and Wang (2016), authors designed two control methods including the adaptive control and state feedback control, then some sufficient conditions to achieve fixed-time stability and synchronization were obtained. In Hua, Li, and Guan (2017), the author designed a new decentralized finite time controller and constructed a fixed-time controller, some criteria were derived to guarantee the finite time stability and fixed-time stability through the recursive method. In Wan, Cao, Wen, and Yu (2016), authors investigated the fixed-time synchronization of CGNNs with parameter uncertainties and time-varying delays, via applying the Filippov discontinuous theory and discontinuous control criteria, some sufficient schemes were obtained to ensure synchronization. As far as authors' knowledge, the finite time and fixed-time synchronization are still absent from the research on CGNNs. Especially, few authors consider the finite time and fixed-time synchronization of CGNNs with discontinuous activation functions via discontinuous control.

Motivated by the above discussions, the main work in this paper is to investigate exponential synchronization, finite time synchronization, and fixed-time synchronization of a class of CGNNs with discontinuous activation functions and time-varying delays. Roughly stated, the main contributions of this paper are reflected as follows: 1. Two controllers, which include discontinuous feedback control and novel adaptive control, are proposed. 2. When the parameters ω in the controller design in this paper take different values, drive–response system can realize exponential synchronization, finite time synchronization and fixed-time synchronization, respectively. This has not been investigated in the existing literatures, which is the main contribution of this paper. 3. By utilizing the Filippov discontinuity theories, the measurable selection theorem and Lyapunov functions, some sufficient conditions are established. 4. The settling time of the fixed-time synchronization derived in this paper is less conservative and more accurate. Actually, the proposed method in this paper will also be applied to the general nonlinear systems. Furthermore, some examples are provided to show the effectiveness and flexibility of the theoretical results.

The rest of this paper is organized as follows. In Section 2, some necessary preliminaries and model description are given. In Section 3, the synchronization of CGNNs with discontinuous activations and time-varying delays is considered and some sufficient conditions are derived based on state feedback control and novel adaptive feedback control, respectively. In Section 4, the flexibility and effectiveness of the theoretical results are shown by some numerical simulations than reported results. Finally, conclusions are drawn in Section 5.

2. Model description and preliminaries

In this paper, we consider the following dynamics of CGNNs with time-varying delays described by

$$\dot{x}_i(t) = -d_i(x_i(t))\Big[a_i(x_i(t)) - \sum_{j=1}^{n} b_{ij} f_j(x_j(t)) - \sum_{j=1}^{n} c_{ij} g_j(x_j(t-\tau(t))) - I_i\Big], \tag{1}$$

where $i \in Z = \{1, 2, \ldots, n\}$, $n \geq 2$ corresponds to the number of units in a neural network, $x_i(t)$ denotes the state variable of the ith neuron at time t, $d_i(\cdot)$ denotes an amplification function, $a_i(\cdot)$ is an appropriately behaved function, b_{ij} and c_{ij} denote the connection strengths of the jth neuron on the ith neuron, respectively. $f_j(\cdot)$ and $g_j(\cdot)$ denote the activation functions, $\tau(t)$ corresponds to the time varying delays result from the finite speed of the axonal signal transmission and satisfies $0 \leq \tau(t) \leq \tau$, τ is a constant. I_i is the input from outside of the networks.

System (1) is supplemented with initial values given by

$x_i(s) = \varphi_i(s), i \in Z, s \in [-\tau, 0]$.

Consider CGNNs (1) as the drive system, suppose that the response system is described by

$$\dot{y}_i(t) = -d_i(y_i(t))\Big[a_i(y_i(t)) - \sum_{j=1}^{n} b_{ij} f_j(y_j(t)) - \sum_{j=1}^{n} c_{ij} g_j(y_j(t-\tau(t))) - I_i - U_i(t)\Big], \tag{2}$$

where $i \in Z$, $y_i(t)$ denotes the state variable of the ith neuron at time t, the other notations are the same as in system (1), $U_i(t)$ is the control input to be designed later.

System (2) is supplemented with initial values given by

$y_i(s) = \psi_i(s), i \in Z, s \in [-\tau, 0]$,

In order to derive the main results, we introduce the following assumptions for all $i \in Z$.

(H_1): $d_i(x)$ is continuous and there exist positive constants $\underline{d}_i$ and $\bar{d}_i$ such that

$$0 < \underline{d}_i \leq d_i(x) \leq \bar{d}_i, \quad x \in R.$$

(H_2): There exist positive constants a_i such that

$$\frac{a_i(y) - a_i(x)}{y - x} \geq a_i, \quad x, y \in R.$$

(H_3): For every $j \in Z$, f_j and g_j are continuous except on a countable set of isolate points $\{\rho_k^j\}$, where the right and left limits $f_j^+(\rho_k^j)$, $f_j^-(\rho_k^j)$, $g_j^+(\rho_k^j)$ and $g_j^-(\rho_k^j)$ exist.

Remark 1. Note that the dynamics of CGNNs (1) and (2) are discontinuous, which is different from the continuous dynamics of the traditional neural networks. In fact, discontinuity phenomenon of activation function is usually encountered since neural networks often inevitably suffer environment noises, limitations of equipment, and systems oscillating (Cai, Huang, & Zhang, 2015; Cortes, 2008; Forti & Nistri, 2003) which can cause the parameters varying within small scopes during the modeling. Therefore, the neural networks with discontinuous activation functions and time-varying delays could be further accord with the practical situation (Jawaadaa, Noorani, & Al-sawalha, 2012; Liang, Gong, & Huang, 2016; Yang & Cao, 2013a).

Due to the presence of discontinuous activation functions $f_j(\cdot)$ and $g_j(\cdot)$, systems (1) and (2) are discontinuous, hence, we need to introduce the framework of Filippov in considering the solution of discontinuous right-hand side.

Definition 1 (*Abdurahman & Jiang, 2016*)**.** Consider a system with discontinuous right-hand side

$$\dot{x}(t) = f(t, x_t), \quad t \geq 0, \tag{3}$$

where x_t is defined by $x_t(\theta) = x(t+\theta), \theta \in [-\tau, 0]$ for any $t > 0$, $x_t \in C = \mathcal{C}([-\theta, 0], R^n)$, $f(t, x_t) : R \times C \to R^n$ is Lebesgue measurable and locally essential bounded. A vector function $x(t)$ is called the Filippov solution of initial value problem (3) on $[-\tau, T)$,