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Design of nonlinear optimal control for chaotic synchronization of coupled stochastic neural networks via Hamilton–Jacobi–Bellman equation

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ABSTRACT

This paper presents a new theoretical design of nonlinear optimal control on achieving chaotic synchronization for coupled stochastic neural networks. To obtain an optimal control law, the proposed approach is developed rigorously by using Hamilton–Jacobi–Bellman (HJB) equation, Lyapunov technique, and inverse optimality, and hence guarantees that the chaotic drive network synchronizes with the chaotic response network influenced by uncertain noise signals. Furthermore, the paper provides four numerical examples to demonstrate the effectiveness of the proposed approach.

1. Introduction

The past two decades have witnessed immense innovations in building artificial computational systems around real-world applications, among which neural networks have become one of the most promising systems deployed in numerous scientific areas. In neural network applications, many successful ones rely on the dynamic behaviors of implementable neural networks, and thus the study of many important facets of neural networks, such as stability analysis and periodic solutions, is introduced.

Synchronization of chaotic systems is critical to applications in many fields such as secure communications, chemical reaction, biological systems, information science, brain science, electronics, and so on. Two decades ago Pecora and Carroll (1990) started their pioneering work on the synchronization of chaotic systems which has since made enormous advances. At the same time, Aihara, Takabe, and Toyoda (1990) in 1990 introduced the model of chaotic neural networks, and since then many achievements and applications have been reported in the literature. Among them, not only has the study of synchronization of chaotic neural networks been attracting a lot of attention from researchers (Balasubramaniam, Chandran, & Theesar, 2011; Balasubramaniam & Vembarasan, 2012; Cao, Li, & Ho, 2005; Cao & Lu, 2006; Cui & Lou, 2009; Gan, Xu, & Kang, 2011; Gao, Zhong, & Gao, 2009; Gong, Lewis, Wang, & Xu, 2016; He & Cao, 2008; Huang & Feng, 2009; Huang, Lam, Cao, & Xu, 2007; Li, Fei, & Zhang, 2008; Li & Bohner, 2012; Lin & Wang, 2010, 2011; Lin, Wang, Nian, & Zhang, 2010; Lu, Ho, Cao, & Kurths, 2011; Rastovic, 2011; Sun, Shen, Yin, & Xu, 2013; Sun, Wang, Wang, & Shen, 2016; Sun, Wu, Cui, & Wang, 2017; Wang & He, 2008; Wang & Song, 2009; Wang & Wang, 2007; Wu, Wen, & Zeng, 2012; Wu, Zeng, Zhu, & Zhang, 2011; Yang & Cao, 2007; Yang, Cao, Long, & Rui, 2010; Yang, Cao, & Yang, 2013; Yoshida, Kurata, Li, & Nara, 2010; Yu, Hu, Jiang, & Teng, 2012; Zhang, Wang, & Lin, 2017; Zhu & Cao, 2010), especially in the area of complex networks, but also a wide variety of approaches have been developed to control the chaotic synchronization of neural networks. Such applications include activation feedback control (Wang & Song, 2009), sliding mode control (Lin & Wang, 2010), time-delay feedback approach (Cao et al., 2005), adaptive control (Yang, Cao et al., 2010), impulsive control (Lu et al., 2011), linear coupling method (Wang & Wang, 2007), linear separation method (Wang & He, 2008), pinning control (Yang et al., 2013), compound synchronization (Sun et al., 2013), etc. However, the aforementioned works focused primarily on deterministic neural networks. And when the authors constructed the mathematical models of their neural networks, they did not include the noise process, which is fraught with signal transmission particularly in very large-scale electronic systems.

In practical applications, the presence of disturbance is unavoidable, and a real system is usually influenced by noises of both internal and external random perturbations. Hence, when artificial neural networks are implemented, noise is inevitable and

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should be taken into consideration. During the past few years, research community has started not only paying attention, but also developing a number of useful approaches for the control of chaotic synchronization of stochastic neural networks, which can be found in Bao, Park, and Cao (2016), Huang and Yang (2009), Sakthivel, Samidurai, and Anthoni (2010), Sheng and Zeng (2017), Wan, Cao, and Wen (2017), Wan and Zhou (2011), Wang, She, Zhong, and Cheng (2017), Yang, Zhang, and Shi (2010) and Yu and Cao (2007). In the available literature on control research, almost all of the methods are based on Lyapunov functional together with linear matrix inequality (LMI) in general, while there seems no study on the chaotic synchronization of coupled stochastic neural networks approached from the perspective of nonlinear optimal control assisted by Hamilton–Jacobi–Bellman (HJB) equation.

In the society of control engineering, there is always a strong motivation toward designing optimal control for control systems because such systems automatically have many desirable properties, such as stability, robustness, reduced sensitivity, etc. (Moylan & Anderson, 1973). Furthermore, research results have shown that optimal control can be a most promising approach in modeling biologically-inspired neural networks (Todorov, 2006, Chapter 12; Werbos, 2009). However, with respect to chaotic synchronization of coupled neural networks there still lacks researches on developing optimal control on it. Even though a recent study (Liu, 2009) reported the development of synchronization for chaotic neural networks by an optimal control law, the author did not consider the noise factor. Hence the reported model is a deterministic neural network model, instead of a stochastic one. In addition, the controller developed by the author was constructed by using linear matrix inequality (LMI) approach. To the best of our knowledge, no publication has presented study results on using nonlinear optimal control for chaotic synchronization of coupled stochastic neural networks via solving a Hamilton–Jacobi–Bellman (HJB) equation, which is simpler than using linear matrix inequality (LMI) approach.

From the above discussions about the researches of control approaches for chaotic synchronization of coupled neural network, the available research results lack at least one facet of the following list: noise factor, stochastic feature, nonlinear optimal control option, and HJB equation. To overcome the impact of overlooked facet(s), we developed a new nonlinear optimal control method in this paper for chaotic synchronization of coupled stochastic neural networks. Our nonlinear optimal control is derived from the technique of inverse optimality, which was established by Kalman (1964) for linear systems and by Moylan (1974) for nonlinear systems. The latest development in inverse optimality approach is done by Freeman and Kokotovic (1996), and has further been extended by several researchers to certain nonlinear systems (Krstic & Deng, 1998). In our research, our nonlinear optimal control was rigorously developed, and hence our chaotic response network influenced by uncertain noise signals is shown to be guaranteed to synchronize with the chaotic drive network.

Our main contributions in this paper are summarized as follows: (1) for the first time in the literature, a nonlinear optimal control is developed for chaotic synchronization of coupled stochastic neural networks via solving a Hamilton–Jacobi–Bellman (HJB) equation. This proposed method is simpler than using linear matrix inequality (LMI) approach. (2) besides the derivation of the nonlinear optimal control, a state feedback stabilizing control is proposed to guarantee that the stochastic error system of coupled neural networks achieves global asymptotic stability in probability, which directly leads to the optimal results of that the trajectory of the chaotic response system influenced by heavy noise signals asymptotically approaches the target trajectory of the chaotic drive system, that is, both are in chaotic synchronization. (3) Four simulation examples are provided to demonstrate the effectiveness and the advantages of the proposed approach compared with other existing research results. Among three of them, all response networks are embedded in heavy noises, that is, the magnitude of noises is greater than the oscillation magnitude of one of the network states. This important feature is not presented in most previous studies in the literature.

The rest of the paper is organized as follows. In Section 2, we present the problem formulation and mathematical preliminaries. In Section 3, we detail the theoretical results. In Section 4, we demonstrate the performance of our design with four numerical examples. And finally, we give the conclusion of the paper in Section 5.

2. Problem formulation and preliminaries

In this paper, we consider the following chaotic neural networks as the drive system:

$$dx_i(t) = \left(-\lambda_i x_i(t) + \sum_{j=1}^{n} w_{ij}^1 \widetilde{f}_j(x_j(t)) + \sum_{j=1}^{n} w_{ij}^2 \widetilde{g}_j(x_j(t-\tau)) + I_i \right) dt \tag{1}$$

where $i = 1, 2, \ldots, n$. Mathematically, this can be described by the following matrix–vector compact form:

$$dx(t) = \left(-Ax(t) + W_1\widetilde{f}(x(t)) + W_2\widetilde{g}(x(t-\tau)) + I\right) dt \tag{2}$$

where $x(t) \in R^n$ is the state of the drive neural network, $I \in R^n$ is the input, $A = diag(\lambda_1, \ldots, \lambda_i \cdots, \lambda_n) \in R^{n\times n}$, both $\widetilde{f}(x(t)) = \left[\widetilde{f}_1(x_1(t)), \ldots, \widetilde{f}_j(x_j(t)), \ldots, \widetilde{f}_n(x_n(t))\right]^T \in R^n$ and $\widetilde{g}(x(t-\tau)) = \left[\widetilde{g}_1(x_1(t-\tau)), \ldots, \widetilde{g}_j(x_j(t-\tau)), \ldots, \widetilde{g}_n(x_n(t-\tau))\right]^T \in R^n$ represent the activation functions of neurons, in addition, both $\widetilde{f}_j(x_j(t))$ and $\widetilde{g}_j(x_j(t-\tau))$ are sigmoidal functions that are scalar ones, $W_1 \in R^{n\times n}$ and $W_2 \in R^{n\times n}$ are weight matrices, $\tau \in R^+$ is the time delay.

Based on the concept of drive-response chaotic systems, the corresponding response system of (2) is given as follows:

$$dy(t) = \left(-Ay(t) + W_1\widetilde{f}(y(t)) + W_2\widetilde{g}(y(t-\tau)) + I\right) dt + u(t)dt + \sigma(t, e(t), e(t-\tau))dw(t) \tag{3}$$

where $u(t) \in R^n$ is the control signal, $e(t) = y(t) - x(t)$ is the error state, $w(t) \in R^n$ is an n-dimensional independent standard Wiener process, and $\sigma(t, e(t), e(t-\tau)) \in R^{n\times n}$ is the noise intensity function, both represent random perturbations, i.e., noise signals, in the system.

Our design goal is to develop a nonlinear optimal control $u(t)$ to guarantee that the chaotic response system influenced by uncertain noise signals will be synchronized with the chaotic drive system.

Let us subtract (2) from (3), which yield the following stochastic error system:

$$de(t) = (-Ae(t) + W_1 f(e(t)) + W_2 g(e(t-\tau)))\, dt + u(t)dt + \sigma(t, e(t), e(t-\tau))dw(t) \tag{4}$$

where $f(e(t)) = \widetilde{f}(y(t)) - \widetilde{f}(x(t)) = \widetilde{f}(x(t)+e(t)) - \widetilde{f}(x(t))$ and $g(e(t-\tau)) = \widetilde{g}(y(t-\tau)) - \widetilde{g}(x(t-\tau)) = \widetilde{g}(x(t-\tau)+e(t-\tau)) - \widetilde{g}(x(t-\tau))$.

Remark 2.1. The model (2) represents a very general neural network model which includes the popular Hopfield neural networks, the paradigm of cellular neural networks, the bio-directional associative memory networks, memristor-based recurrent neural networks, and several other neural networks frequently employed in the literature.