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Reduced-order state estimation of delayed recurrent neural networks

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ABSTRACT

Different from the widely-studied full-order state estimator design, this paper focuses on dealing with the reduced-order state estimation problem for delayed recurrent neural networks. By employing an integral inequality, a delay-dependent design approach is proposed, and global asymptotical stability of the resulting error system is guaranteed. It is shown that the gain matrix of the reduced-order state estimator is determined by the solution of a linear matrix inequality. Numerical examples are provided to illustrate the effectiveness of the developed result.

1. Introduction

Compared with feedforward neural networks (Qian, Huang, Chen, & Huang, 2017), recurrent neural networks (RNNs) generally have more complicated dynamical behaviors due to their self-evolutions with time (Haykin, 1999). Consequently, RNNs have gained many successful applications in different areas including combinatorial optimization, signal processing, wireless communication and intelligent control, etc.

To achieve better performance for some practical problems, time delay is intentionally introduced in the models of RNNs. On the other hand, time delay is also encountered in RNNs because of the signal transmission between different neurons. However, as a disadvantage, the presence of time delay would greatly change the dynamical behavior of the underlying RNN such that the stability property would be destroyed. Therefore, stability and passivity analysis and synchronization of delayed RNNs have been extensively studied in the past few years (Anbuvithya, Mathiyalagan, Sakthivel, & Prakash, 2016; Ding, Wang, Huang, & Zhang, 2017; Huang, Huang, & Chen, 2012; Huang, Li, Duan, & Starzyk, 2012; Huang, Li, Yu, & Chen, 2009; Kwon, Park, Lee, & Cha, 2014; Mathiyalagan, Anbuvithya, Sakthivel, Park, & Prakash, 2016; Wang, Ding, Shan, & Zhang, 2017; Yang, Wang, & Wang, 2017; Zhang, He, Jiang, & Wu, 2016). A complete survey on stability of delayed RNNs can be found in Zhang, Wang, and Liu (2014). In Yang et al. (2017) and Zhang et al. (2016), some integral inequalities were proposed to discuss global asymptotical stability of delayed RNNs such that less conservative stability conditions were obtained.

Recently, the state estimation problem of delayed RNNs has attracted considerable attention (Wang, Ho, & Liu, 2005). This is because, in a RNN with a great number of neurons, it is very hard or even impossible to get the state information of all neurons. While, these information are often useful for practical applications. Therefore, the study of the state estimator design for different RNNs with time delays is of great significance. Sufficient design criteria were reported in the literature (see, e.g., Ding, Wang, Wang, and Zhang (2016); Hou, Dong, and Wang (2017); Huang, Feng, and Cao (2008); Huang, Huang, and Chen (2013); Huang, Huang, and Chen (2015); Mathiyalagan, Su, Shi, and Sakthivel (2015); Ratnavelu, Manikandan and Balasubramaniam (2017); Sakthivel, Anbuvithya, Mathiyalagan, and Prakash (2015); Wang, Wang, and Wu (2017); Xu, Lu, Peng, Xie, and Xue (2017); Xu, Lu, Shi, Tao, and Xie (in press)). It should be noted that most of the results mentioned above are concerned with the full-order state estimator design of delayed RNNs. While, in practice, the state information of some neurons in a RNN may be measured. For these states, it is no longer needed to be estimated. In addition, there are many circumstances that only the information of a certain part of neurons is required to be utilized for the problems in-hand. That is to say, it is not always necessary to estimate the state information of all neurons. At the same time, the computational cost would be very high when all neurons' states are estimated, especially for a delayed RNN with a great number of neurons. This will be shown later. It is thus worth studying the reduced-order state estimation problem for delayed RNNs. To our knowledge, this issue has not yet been investigated.

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Motivated by these observations, this paper is dedicated to resolving the reduced-order state estimator design problem for a class of delayed RNNs. By constructing an appropriate Lyapunov–Krasovskii functional and employing an integral inequality, a sufficient condition in terms of linear matrix inequality (LMI) is derived under which the resulting error system is globally asymptotically stable. Then, a desired reduced-order state estimator is designed via the feasible solution of the corresponding LMI and thus can be efficiently implemented in practice. Two numerical examples are provided to show the application and effectiveness of the developed design approach. The main contributions and advantages of this study lie in three-folds: (i) it is the first time to study the reduced-order state estimation problem for delayed RNNs and an efficient approach is proposed to solve it; (ii) compared with the full-order one, it is shown that the computational cost required in the implementation of the reduced-order state estimator is heavily reduced; and (iii) by employing the property of Legendre polynomials, an integral inequality is presented to reduce the conservativeness of the design criterion.

2. Problem formulation

Some notations used in this paper are defined here for convenience. $\mathscr{R}^n$ represents the n-dimensional Euclidean space and $\mathscr{R}^{n\times m}$ is the set of all $n \times m$ real matrices. I and $\mathbf{0}$ are, respectively, the identity and zero matrices with compatible dimensions. The transpose of a vector or matrix is denoted by the superscript "T". For a real matrix $X \in \mathscr{R}^{n\times n}$, X^{-1} is the inverse of X if applicable, and $X > 0\,(X < 0)$ means that X is symmetric and positive definite (negative definite).

The delayed RNN discussed in this study is described by

$$\dot{x}(t) = -Ax(t) + Wf(x(t)) + W_d f(x(t-d)) + J \tag{1}$$

$$y(t) = Cx(t) \tag{2}$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in \mathscr{R}^n$ and $y(t) \in \mathscr{R}^m$ are, respectively, the state vector of the delayed RNN (1) and available measured output, $A = \mathrm{diag}(a_1, a_2, \ldots, a_n)$ is a diagonal matrix with $a_i > 0$ representing the firing rate of neuron i ($i = 1, 2, \ldots, n$), $W \in \mathscr{R}^{n\times n}$ and $W_d \in \mathscr{R}^{n\times n}$ are, respectively, the connection weight matrix and delayed connection weight matrix, $f(\cdot) = [f_1(\cdot), f_2(\cdot), \ldots, f_n(\cdot)]^T$ is an activation function, $J \in \mathscr{R}^n$ denotes the external input, $d > 0$ is a known constant which represents the time delay of the delayed RNN (1), and $C \in \mathscr{R}^{m\times n}$ is a constant matrix.

The activation function $f(\cdot)$ is assumed to satisfy the following condition:

Assumption 1. There are positive scalars l_i ($i = 1, 2, \ldots, n$) such that

$$0 \le \frac{f_i(a) - f_i(b)}{a - b} \le l_i. \tag{3}$$

Furthermore, it is assumed that the matrix C in (2) is of full row rank.

As discussed in Section 1, it is usually very difficult to obtain the complete information of neurons of the delayed RNN (1). The full-order state estimation problem for delayed RNNs was thus extensively investigated in recent years (see, e.g., Ding et al. (2016); Hou et al. (2017); Huang et al. (2008); Huang et al. (2013); Huang et al. (2015); Mathiyalagan et al. (2015); Ratnavelu et al. (2017); Sakthivel et al. (2015); Wang, Wang et al. (2017); Xu, Lu, Shi et al. (in press)). However, in many practical applications, it is not necessary to estimate the information of all neurons since it would be very expensive. In this circumstance, it should be more suitable to design a reduced-order state estimator for the delayed RNN (1). That is, instead of estimating the full state $x(t)$, only a partial state $u(t)$ of $x(t)$ is required to be estimated, which is defined as

$$u(t) = Hx(t) \tag{4}$$

where $u(t) \in \mathscr{R}^{n-m}$ and $H \in \mathscr{R}^{(n-m)\times n}$ is a known matrix with full row rank. Since C is also of full row rank, one can easily choose H such that the matrix $\begin{bmatrix} C \\ H \end{bmatrix}$ is nonsingular. For example, when $C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$, the matrix H can be chosen as $H = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. It means that, for the delayed RNN (1), $x_1(t)$ is available and one only needs to estimate $x_2(t)$ and $x_3(t)$.

Since $\begin{bmatrix} C \\ H \end{bmatrix}$ is invertible, one can find two matrices $M \in \mathscr{R}^{n\times m}$ and $N \in \mathscr{R}^{n\times(n-m)}$ such that

$$\begin{bmatrix} C \\ H \end{bmatrix}^{-1} = \begin{bmatrix} M & N \end{bmatrix}.$$

It is known from (2) and (4) that

$$x(t) = My(t) + Nu(t). \tag{5}$$

This together with (1) and (4) gives

$$\begin{aligned} \dot{u}(t) = &-HAMy(t) - HANu(t) + HWf(My(t) + Nu(t)) \\ &+ HW_d f(My(t-d) + Nu(t-d)) + HJ. \end{aligned} \tag{6}$$

According to (6), a reduced-order state estimator for the delayed RNN (1) is constructed as

$$\begin{aligned} \dot{\bar{u}}(t) = &-HAMy(t) - HAN\bar{u}(t) + HWf(My(t) + N\bar{u}(t)) \\ &+ HW_d f(My(t-d) + N\bar{u}(t-d)) + HJ + K(\dot{y}(t) - \hat{u}(t)) \end{aligned} \tag{7}$$

where $K \in \mathscr{R}^{(n-m)\times m}$ is a gain matrix to be designed, and

$$\begin{aligned} \hat{u}(t) = &-CAMy(t) - CAN\bar{u}(t) + CWf(My(t) + N\bar{u}(t)) \\ &+ CW_d f(My(t-d) + N\bar{u}(t-d)) + CJ. \end{aligned}$$

Define the error between $u(t)$ and $\bar{u}(t)$ as $\varepsilon(t) = u(t) - \bar{u}(t)$. It follows from (6) and (7) that

$$\begin{aligned} \dot{\varepsilon}(t) = &-(A_H - KA_C)\varepsilon(t) + (W_H - KW_C)g(N\varepsilon(t)) \\ &+ (W_{dH} - KW_{dC})g(N\varepsilon(t-d)), \end{aligned} \tag{8}$$

where $A_H = HAN, A_C = CAN, W_H = HW, W_C = CW, W_{dH} = HW_d, W_{dC} = CW_d$ and

$$g(N\varepsilon(t)) = f(My(t) + Nu(t)) - f(My(t) + N\bar{u}(t)).$$

Let $g(\cdot) = [g_1(\cdot), g_2(\cdot), \ldots, g_n(\cdot)]^T$. From (3), it is obvious that, for $i = 1, 2, \ldots, n$, $g_i(0) = 0$ and

$$0 \le \frac{g_i(v)}{v} \le l_i. \tag{9}$$

Then, for diagonal matrices $\Lambda = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_n) > 0$ and $\Sigma = \mathrm{diag}(\sigma_1, \sigma_2, \ldots, \sigma_n) > 0$, one has

$$\begin{aligned} 0 &\le -2\sum_{i=1}^{n} \lambda_i g_i(N_i\varepsilon(t))(g_i(N_i\varepsilon(t)) - l_i N_i\varepsilon(t)) \\ &= -2g^T(N\varepsilon(t))\Lambda g(N\varepsilon(t)) + 2g^T(N\varepsilon(t))\Lambda LN\varepsilon(t), \end{aligned} \tag{10}$$

$$\begin{aligned} 0 \le &-2g^T(N\varepsilon(t-d))\Sigma g(N\varepsilon(t-d)) \\ &+ 2g^T(N\varepsilon(t-d))\Sigma LN\varepsilon(t-d), \end{aligned} \tag{11}$$

where $L = \mathrm{diag}(l_1, l_2, \ldots, l_n)$ and N_i is the ith row of N.

Remark 1. The objective of this study is to present an efficient approach to tackle the reduced-order state estimation problem for