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Impact of leakage delay on bifurcation in high-order fractional BAM neural networks[☆]

Chengdai Huang[a,*], Jinde Cao[b,c]

[a] School of Mathematics and Statistics, Xinyang Normal University, Xinyang 464000, China
[b] School of Mathematics, and Research Center for Complex Systems and Network Sciences, Southeast University, Nanjing 210996, China
[c] School of Mathematics and Statistics, Shandong Normal University, Ji'nan 250014, China

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ABSTRACT

The effects of leakage delay on the dynamics of neural networks with integer-order have lately been received considerable attention. It has been confirmed that fractional neural networks more appropriately uncover the dynamical properties of neural networks, but the results of fractional neural networks with leakage delay are relatively few. This paper primarily concentrates on the issue of bifurcation for high-order fractional bidirectional associative memory(BAM) neural networks involving leakage delay. The first attempt is made to tackle the stability and bifurcation of high-order fractional BAM neural networks with time delay in leakage terms in this paper. The conditions for the appearance of bifurcation for the proposed systems with leakage delay are firstly established by adopting time delay as a bifurcation parameter. Then, the bifurcation criteria of such system without leakage delay are successfully acquired. Comparative analysis wondrously detects that the stability performance of the proposed high-order fractional neural networks is critically weakened by leakage delay, they cannot be overlooked. Numerical examples are ultimately exhibited to attest the efficiency of the theoretical results.

1. Introduction

Over the last few decades, the dynamical properties of neural networks have been highly concerned (Balasundaram, Raja, Zhu, Chandrasekaran, & Zhou, 2016; Basu et al., 2018; Cao & Li, 2017; Cao, Rakkiyappan, Maheswari, & Chandrasekar, 2016; Dosovitskiy, Fischer, Springenberg, Riedmiller, & Brox, 2016; Huang, Huang, & Chen, 2013, 2012, 2015; Kumar, Sugumaran, Raja, Zhu, & Raja, 2016; Li & Cao, 2016; Senthilraj, Raja, Zhu, Samidurai, & Yao, 2016; Tu, Cao, Alsaedi, & Hayat, 2017; Wan, Cao, & Wen, 2017; Wen, Chen, Liu, & Liu, 2017; Wu, Cao, Li, Alsaedi, & Alsaadi, 2017) owing to their successful utilization in optimization, signal processing, associative memory, parallel computation, pattern recognition, artificial intelligence, et al. The fixed-time synchronization of a class of delayed memristive neural networks is meticulously considered, and some easily-verified conditions were derived to achieve fixed time synchronization for such neural networks (Cao & Li, 2017). The global dissipativity of quaternion-valued neural networks with time-varying delays was firstly examined, and the algebraic criteria ensuring the global dissipativity and globally exponential dissipativity for such neural networks were put forward based on Lyapunov theory and analytic techniques (Tu et al., 2017). The low-order neural networks possess many immanent limitations in convergence rate, storage capacity, and fault tolerance. Neural networks with high-order admit palpable advantages involving bigger storage capacity, stronger approximation property, faster convergence rate, and higher fault tolerance in comparison with lower-order ones. This emboldens numerous researchers to take advantage of neural networks with high order connections. Thus, it is essential and momentous to analyze high-order interactions to neural networks for overcoming the shortcomings of low-order neural networks. Up to present, a number of eximious results on the dynamics of high-order neural networks have been gained (Huang, Cao, Alofi et al., 2017; Huang, Cao, Xiao, Alsaedi, & Hayat, 2018; Nie & Huang, 2012; Ren & Cao, 2007).

Fractional calculus has been applied proverbially in substantial disciplines (Kilbas, Srivastava, & Trujillo, 2006; Koeller, 1984; Matignon, 1996). The cardinal reason is that fractional derivatives afford an effective, commendable artifice for the description of memory and genetic properties of diverse materials and operations compared with integer-order derivatives. It is revealed that fractional-order dynamical systems can offer more accurate results than their integer-order counterparts. Accordingly, it is

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* Corresponding author.
E-mail address: huangchengdai@163.com (C. Huang).

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imperative and vital to investigate fractional-order dynamical systems theoretically and applicably. With the rapid development of fractional calculus, some researchers amazingly discovered that fractional calculus can be incorporated into neural networks and fractional order version neural networks are developed, which is able to reflect the dynamical properties with more precision in virtue of their infinite memory property. Recently, a great deal of remarkable results on dynamical behaviors for fractional order neural networks have been reported (Chen, Wu, Cao, & Liu, 2015; Huang, Cao, Xiao, Alsaedi, & Hayat, 2017; Huang et al., 2018; Huang, Meng, Cao, Alsaedi, & Alsaadi, 2017; Rajivganthi, Rihan, Lakshmanan, & Muthukumar, 2016; Rakkiyappan, Cao, & Velmurugan, 2015; Rakkiyappan, Velmurugan & Cao, 2015; Velmurugan, Rakkiyappan, & Cao, 2016; Wang, Yu, & Wen, 2014; Xiao, Zheng, Jiang, & Cao, 2015; Zhang, Song, & Zhao, 2017). Huang et al. (2018) studied the problem of bifurcation for a class of high-dimension fractional ring-structured neural networks with multiple time delays, and the impact of time delay, the order and the number of neurons on the stability and bifurcation of the proposed networks was detailedly discussed. The issue of finite-time synchronization of fractional-order memristor-based neural networks was considered, the results of finite-time synchronization of such networks were derived (Velmurugan et al., 2016). Rajivganthi et al. (2016) discussed the problem of finite-time stability for a class of fractional-order delayed Cohen–Grossberg BAM neural networks based on the inequality techniques, differential mean value theorem and contraction mapping principle, and sufficient conditions were established to ensure the finite-time stability of the presented neural networks. The uniform stability for fractional-order complex-valued neural networks with both leakage and discrete delays was analyzed based on analysis technique, and delay-dependent criteria were derived to guarantee global uniform stability for such neural networks (Zhang, Song, & Zhao, 2017).

Kosko (1987) pioneered the BAM networks, these networks are comprised of neurons arranged in two layers: the X-layer and the Y-layer. Thanks to their peculiar structures, the BAM networks have much prominent performance including the paired pattern or memories of storage and the capability of searching the desired patterns through two directions: forward and backward. BAM networks have been applied in various areas, such as, pattern recognition (Lee, 1998), signal and image processing, etc. The delayed BAM neural networks were developed (Gopalsamy & He, 1994). After then, numerous good results on the dynamics of BAM neural networks have been obtained (Ali, Balasubramaniam, & Zhu, 2017; Ratnavelua, Manikandana, & Balasubramaniam, 2015; Samidurai, Senthilraj, Zhu, Raja, & Hu, 2017; Sayli & Yilmaz, 2014; Senan & Arik, 2009; Senan, Arik, & Liu, 2012; Ye, Zhang, Zhang, Zhang, & Lu, 2015; Zhu, Li & Li, 2011). The authors discussed the mean square exponential stabilization problem of stochastic BAM neural networks with Markovian jumping parameters and time-varying delays in terms of Lyapunov–Krasovskii functional and LMIs technique, some sufficient conditions were derived for ensuring exponential stabilization in the mean square sense of such networks (Ye, Zhang, Zhang, Zhang, & Lu, 2015).

Afterwards, the BAM neural networks were triumphantly popularized to the versions ones with leakage delays, and further spotted that the stability performance can be heavily undermined by leakage delays for such neural networks (Gopalsamy, 2007). Recently, a number of outstanding results on the dynamics for the influence of leakage delays in BAM networks have been derived (Lakshmanan, Ju, Lee, Jung, & Rakkiyappan, 2013; Sakthivel, Vadivel, Mathiyalagan, Arunkumar, & Sivachitra, 2015; Samidurai et al., 2017; Wang & Liu, 2016; Zhu, Rakkiyappan, & Chandrasekar, 2014). In Sakthivel et al. (2015), sufficient conditions which guarantee the state estimation of the BAM neural networks with leakage delay, constant distributed and time-varying discrete delays obtained, and it revealed that leakage delays in the proposed neural networks cannot be ignored. It is a great pity that fractional calculus was not introduced into these BAM network. In recent years, the influence of leakage delay on fractional BAM neural networks has received increasing attention of many researchers. The uniform stability for a fractional BAM neural networks with leakage delays was examined (Yang, Song, Liu, & Zhao, 2014). Cao and Bai (2015) discussed the existence and uniqueness of the nontrivial solution for fractional delayed BAM neural networks. It is indicated that leakage delays have a tremendous destabilizing effect on the dynamical system and they cannot be disregarded.

There is a forceful instrument for Hopf bifurcation analysis to achieve the dynamical properties of nonlinear systems (Celik & Merdan, 2013; Guo & Yan, 2016; Kmit & Recke, 2014; Ouifki, Hbid, & Arino, 2003). Clearly, Hopf bifurcations of integer-order system have been adequately studied. Lately, some scholars are fascinated by the bifurcation problem of fractional order systems and some valuable efforts have been made, a number of eminent results have been acquired (Huang, Cao, Alofi et al., 2017; Huang, Cao, Xiao et al., 2017; Huang et al., 2018; Xiao et al., 2015). Unfortunately, the impact of leakage delays has been not taken into account in these excellent results. It should be pointed out that very little work has been paid to bifurcation results for fractional BAM neural network with time delay in leakage terms (Huang, Meng et al., 2017). It is clear that the issue of bifurcation for low-order simple neural network model was merely considered (Huang, Meng et al., 2017). It is still unknown whether leakage delay effects the bifurcation of high-order fractional BAM neural networks. To the best of our knowledge, this concern is still open, challenging and has not been appropriately studied.

Stimulated by the above discussions, the primary objective of this paper is to deal with the problem of Hopf bifurcation for high-order fractional BAM networks with leakage delay. The main contributions of this paper can be summarized as follows: (1) The stability and bifurcation in a class of high-order fractional BAM network with time delay in leakage term is firstly discussed. (2) The conditions of the onset of bifurcation for the proposed systems with or without leakage delay are accurately established, respectively. (3) The effects of leakage delay on the dynamics for such high-order fractional BAM networks are sufficiently studied through comparative analyses numerically. It is demonstrated that leakage delay severely disrupts the stability performance for the proposed systems.

The outline of the paper is highlighted as follows: In Section 2, some definitions and Lemmas of fractional calculus are firstly recalled. The discussed mathematical models are proposed in Section 3. In Section 4, the conditions of Hopf bifurcation are precisely established with or without leakage delay, respectively. Our theoretical findings are utterly corroborated by means of numerical examples in Section 5. Conclusions are finally drawn.

2. Preliminaries

In this section, we present some basic definitions and lemmas of fractional calculus which can be useful to derive our main results. The definition of the Caputo fractional-order derivative shall be employed in this paper.

Definition 1 (*Podlubny, 1999*)**.** The fractional integral of order ρ for a function $f(t)$ is defined as

$$I^{\rho}f(t)=\frac{1}{\Gamma(\rho)}\int_{t_0}^{t}(t-s)^{\rho-1}f(s)ds,$$

where $t \geq t_0$, $\rho > 0$, $\Gamma(\cdot)$ is the Gamma function, $\Gamma(s) = \int_0^{\infty} t^{s-1}e^{-t}dt$.