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Region stability analysis and tracking control of memristive recurrent neural network

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ABSTRACT

Memristor is firstly postulated by Leon Chua and realized by Hewlett–Packard (HP) laboratory. Research results show that memristor can be used to simulate the synapses of neurons. This paper presents a class of recurrent neural network with HP memristors. Firstly, it shows that memristive recurrent neural network has more compound dynamics than the traditional recurrent neural network by simulations. Then it derives that n dimensional memristive recurrent neural network is composed of 2^{2n^2} sub neural networks which do not have a common equilibrium point. By designing the tracking controller, it can make memristive neural network being convergent to the desired sub neural network. At last, two numerical examples are given to verify the validity of our result.

1. Introduction

Dynamics of recurrent neural network (RNN) make a solid foundation for its applications in neuron computation, associative memory, pattern recognition and image processing. Extensive neurobiological experiments and theoretical analysis will contribute to understanding and modeling the function of human brain (see Li, Yu, and Huang, 2014; Li, Yu, Huang, and He, 2017, Liao, Luo, and Zeng, 2007, Hu, Yu, Chen, Jiang, and Huang, 2017, Wu, Liu, Huang, and Zeng, 2017 and their references). Very large scale integrated (VLSI) circuit implementation of neural furthers the research of neural networks (Hopfield, 1982). In the VLSI, resistors are used to emulate the synapses of neurons but cannot exhibit the variability of synapses. So a new device for emulating the synapses is needed to improve the neural network circuit.

Fortunately, memristor, the missing fourth device, is arising. It is theoretically postulated by Leon Chua (Chua, 1971) and physically realized by Strukov, Snider, Stewart and Williams in Hewlett Packard (HP) Laboratories (Strukov, Snider, Stewart, & Williams, 2008). The HP memristor is a nano-scale device which is made by sandwiching semiconductor film (TiO_2) between two metal contacts (Pt). Memristance varies with the applied voltage and time. Pershin and Di Ventra show that memristor can work like a biological synapse by associative memory experiment on a dog (Pershin & Di Ventra, 2010). Therefore it becomes the research focus to model neuron networks with memristors.

To the best of our knowledge, there are two research aspects in the existing literatures about memristive recurrent neural network (MRNN). One aspect is that it emulates biological neuron synapse by using digital and analog circuit technology, such as, neuromorphic computation (Pershin & Di Ventra, 2011), memristor bridge synapses (Wang & Shen, 2015; Wang, Shen, Yin, & Zhang, 2015; Wang, Wang, Duan, & Li, 2015), memristor bridge synapse-based neural network (Adhikari, Yang, Kim, & Chua, 2012), neural synaptic weighting with a pulsed-based memristor circuit (Kim, Sah, Yang, Roska, & Chua, 2012), memristor-based neural networks (Thomas, 2013), memristive perceptron, etc.; another aspect is to model neuron networks by using the same circuit structure of Hopfield neural network or cellular neural network just replacing the resistors with memristors. Such as, Itoh and Leon Chua present memristor automata and memristor discrete-time cellular neural networks by assuming memristance with multi-value (Itoh & Chua, 2009). With assumption of memristance switching between two resistance, recurrent neural network with memristors is derived (Hu & Wang, 2013). According to memristance definition of ideal memristor, it gives the model of cellular neural networks with dynamic memristors and second-order cells (Marco, Forti, & Pancioni, 2016). Observing the periodic variation of memristance, neural network with time-varying coefficients is presented and one sufficient condition is derived for such neural network Bao and Zeng (2013).

In order to get successful application of MRNN, researchers have done many works about dynamic properties of MNN, such as,

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passivity analysis (Duan & Huang, 2014; Guo, Wang, & Yan, 2013; Tu, Cao, Alsaedi, & Alsaadi, 2017), attractive analysis (Guo, Wang, & Yan, 2014; Qin, Wang, & Xue, 2015), synchronization analysis (Abdurahman & Jiang, 2016; Abdurahman, Jiang, & Teng, 2015, 2016; Bao, Park, & Cao, 2016; Chen, Wu, Cao, & Liu, 2015; Gao, Zhu, Alsaedi, Alsaadi, & Hayat, 2017; Li & Cao, 2015; Li, Duan, Liu, Wang, & Huang, 2016; Yang, Li, & Huang, 2016; Yang, Luo, Liu, & Li, 2017; Zhang, Li, Huang, & He, 2015), almost periodic analysis (Jiang, Zeng, & Chen, 2015), finite time stability analysis (Wang, Song, Liu, Zhao, & Alsaadi, 2017), and other dynamics analysis (Wang & Shen, 2015; Wen, Bao, Zeng, Chen, & Huang, 2013; Wen, Huang, Zeng, Chen, & Li, 2015; Wen & Zeng, 2015; Wen, Zeng, & Huang, 2013 etc.). In the most of the works, MRNN is modeled as differential equation systems with discontinuous right hand side. By defining solution in Filippov and differential inclusion theory, researchers analyze dynamics of MRNN by constructing suitable Lyapunov functions. All of these results are based on the assumption that memristance is switched between two different values. But according to the memristance model of HP memristor, the value of memristor is a continuous bounded function with respect to time t. Hence, the main work of this paper is presenting a differential equation systems for MRNN, proving 2^{2n^2} sub neural networks with no one common equilibrium point, analyzing region stability of MRNN, and then designing tracking controller to make MRNN be convergent to the desired sub neural network.

The rest of the paper is arranged as the following. In Section 2, it presents a mathematical model of MRNN, some preliminaries and analyzes dynamics by simulations. And then Section 3 studies global asymptotical region stability of MRNN and presents two attractive sets with respect to two different Lyapunov functions. Then it designs tracking controller for MRNN so that it can make MRNN being convergent to the desired sub neural network in Section 4. In the following Section 5, it concludes the work in this paper. Two illustrative examples are given in Section 6.

2. Preliminaries

In this section, we give the mathematical model of MRNN which is inspired by Hopfield neural network model. Then we introduce a numerical example for illustrating dynamics of MRNN by simulation with Matlab.

2.1. Model description

The MRNN can be described by

$$C_i\frac{dx_i(t)}{dt} = -\frac{x_i(t)}{\rho_i} + \sum_{j=1}^{n}\frac{f_j(x_j(t)) - x_i(t)}{R_{ij}} + I_i, \tag{1}$$

where $\rho_i, C_i, R_{ij}\,(i,j = 1,2,\ldots,n)$ are input resistances of amplifier i, total input capacitances of amplifier i, resistance $\frac{1}{R_{ij}}$ representing the strength of synapses, respectively. I_i, $i = 1,2,\ldots,n$, are bias voltages. Functions $f_j(x)$, $j = 1,2,\ldots,n$ are activation functions, and

$$|f_j(r_1) - f_j(r_2)| \le l_j|r_1 - r_2| \tag{2}$$

for $j = 1,2,\ldots,n$, $l_j > 0$ and $L = (l_1, l_2, \ldots, l_n)^T$. $f_j(x_j(t)) - x_i(t)$, $i,j = 1,2,\ldots,n$ are voltages across resistor R_{ij}. According to the common idea in the existing literatures, we replace resistors with memristors, use memristance model, and then derive the math model of MRNN

$$C_i\frac{dx_i(t)}{dt} = -\frac{x_i(t)}{\rho_i} + \sum_{j=1}^{n}\frac{f_j(x_j(t)) - x_i(t)}{M_{ij}(t)} + I_i, \tag{3}$$

where M_{ij} is memristance and modeled as

$$\begin{aligned} M_{ij}(t) &= s(t)R_{on} + (1-s(t))R_{off} \\ \frac{ds(t)}{dt} &= g(s(t), v_m) \end{aligned} \tag{4}$$

in which $s(t)$, i_m, v_m are memristor state, current and voltage, respectively. $g(s(t), v_m)$ is the state evolution function (Ascoli, Tetzlaff, & Chua, 2016). Consider time delays and let $\kappa_i = \frac{1}{\rho_i}$, $a_{ij} = \frac{1}{M_{ij}(t)}$, $b_{ij} = \frac{1}{M'_{ij}(t)}$. Then the general model for MRNN is

$$\begin{aligned} \frac{dx_i(t)}{dt} = -\kappa_i x_i(t) &+ \sum_{j=1}^{n} a_{ij}f_j(x_j(t)) \\ &+ \sum_{j=1}^{n} b_{ij}f_j(x_j(t-\tau)) + I_i \end{aligned} \tag{5}$$

where κ_i, $i = 1,2,\ldots,n$ are positive constants; a_{ij}, b_{ij} are coefficients, I_i are bias voltages; activation functions $f_1(r) = f_2(r) = (|r+1| - |r-1|)/2$, $r \in R^n$.

Remark 1. In the existing literatures, the memristor is simplified as a two-resistance device according to the incline eight type. From the experiment and computation analysis in memristance variation properties are different under DC and AC voltage. Under DC voltage, memristance switches between R_{on} and R_{off} when the $s(t)$ is at stable state. With AC voltage, memristance is a periodic function with respect to time t when the $s(t)$ is at stable state. The variation characteristics are not related to initial values. We discuss dynamics of MRNN under DC voltage. From (5), terminal voltages of memristors are $f_j(x_j(t)) - x_i(t)$, $i,j = 1,2,\ldots,n$. Obviously, $M_{ij}(t) = R_{on}$ or R_{off} if MRNN is at stable state. Based on our team works, we take memristance model as $M_{ij}(t) = s(t)R_{on} + (1 - s(t))R_{off}$, i.e., memristance is continuous variable between R_{on} and R_{off}. MRNN is going to be stable at one of sub neural networks or switching among 2^{2n^2} sub neural networks. This is the difference from the MRNN model described by differential inclusion. For simplifying discussion, hence we assume that coefficients of MRNN as

$$a_{ij} = \begin{cases} \hat{a}_{ij}, & f(x_i(t)) - x_j(t) \ge 0 \\ \check{a}_{ij}, & f(x_i(t)) - x_j(t) < 0 \end{cases}$$

$$b_{ij} = \begin{cases} \hat{b}_{ij}, & f(x_i(t)) - x_j(t) \ge 0 \\ \check{b}_{ij}, & f(x_i(t)) - x_j(t) < 0 \end{cases}$$

for $i,j = 1,2,\ldots,n$ regardless of the effects caused by initial values and dwell time of voltage direction changing. Let

$$\begin{aligned} \tilde{a}_{ij} &= \max\{|\hat{a}_{ij}|, |\check{a}_{ij}|\} \\ \tilde{b}_{ij} &= \max\{|\hat{b}_{ij}|, |\check{b}_{ij}|\} \\ |A|_{\max} &= (\tilde{a}_{ij}), \quad |B|_{\max} = (\tilde{b}_{ij}). \end{aligned}$$

Next, we show the difference between traditional neural network and MRNN by the following numerical simulation example.

Example 1. Considering the following MRNN

$$\begin{cases} \dot{x_1}(t) = -0.4x_1(t) + \sum_{j=1}^{2} a_{1j}f_j(x_j(t)) + 0.02 \\ \dot{x_2}(t) = -0.6x_2(t) + \sum_{j=1}^{2} a_{2j}f_j(x_j(t)) + 0.05 \end{cases} \tag{6}$$

where

$$a_{11} = \begin{cases} 0.3, & f(x_1(t)) - x_1(t) \ge 0 \\ 0.4, & f(x_1(t)) - x_1(t) < 0 \end{cases}$$