Neural Networks 97 (2018) 11–18

Contents lists available at ScienceDirect

Neural Networks

journal homepage: www.elsevier.com/locate/neunet

Neural network robust tracking control with adaptive critic framework for uncertain nonlinear systems[☆]

Ding Wang[a,b,*], Derong Liu[c], Yun Zhang[c], Hongyi Li[c]

[a] The State Key Laboratory of Management and Control for Complex Systems, Institute of Automation, Chinese Academy of Sciences, Beijing 100190, China
[b] School of Computer and Control Engineering, University of Chinese Academy of Sciences, Beijing 100049, China
[c] School of Automation, Guangdong University of Technology, Guangzhou 510006, China

ARTICLE INFO

Article history:
Received 25 February 2017
Received in revised form 18 July 2017
Accepted 4 September 2017
Available online 21 September 2017

Keywords:
Adaptive critic designs
Dynamical uncertainty
Learning systems
Neural networks
Optimal control
Robust tracking control

ABSTRACT

In this paper, we aim to tackle the neural robust tracking control problem for a class of nonlinear systems using the adaptive critic technique. The main contribution is that a neural-network-based robust tracking control scheme is established for nonlinear systems involving matched uncertainties. The augmented system considering the tracking error and the reference trajectory is formulated and then addressed under adaptive critic optimal control formulation, where the initial stabilizing controller is not needed. The approximate control law is derived via solving the Hamilton–Jacobi–Bellman equation related to the nominal augmented system, followed by closed-loop stability analysis. The robust tracking control performance is guaranteed theoretically via Lyapunov approach and also verified through simulation illustration.

1. Introduction

With the development of various promising techniques related to artificial intelligence and computational intelligence, many efforts have been given to the fields of neural networks and learning systems. Among them, the neural-network-based learning control design has attained great attention, especially under general uncertain environment. The robust control problem is traditionally addressed for dynamical systems with uncertainties. The combination of optimal feedback control and robust stabilization has attracted special attention (Lin, 2007). After that, by considering the idea of adaptive critic designs (Werbos, 1992, 2009), some learning-oriented robust control approaches were proposed (Bian, Jiang, & Jiang, 2015; Fan & Yang, 2016; Gao, Jiang, Jiang, & Chai, 2016; Jiang & Jiang, 2014; Sun, Liu, & Ye, 2017; Wang, Liu, Zhang, & Zhao, 2016; Zhong, He, & Prokhorov, 2013). A common property of these results is the introduction of adaptive critic designs, which is an intelligent optimization method involving the idea of reinforcement learning. If we study optimal control problems based on adaptive critic, we should approximate the solution of the Hamilton–Jacobi–Bellman (HJB) equation, which is difficult to address directly. The adaptive optimal control of nonlinear systems has been studied based on adaptive critic (Dierks & Jagannathan, 2010; Luo, Wu, Huang, & Liu, 2015; Lv, Na, Yang, Wu, & Guo, 2016; Song, Lewis, Wei, & Zhang, 2016; Vamvoudakis & Lewis, 2010) with extension to differential game design (Zhao, Zhang, Wang, & Zhu, 2016). As for the dynamical uncertainties, the approximate HJB based solution can be applied to handle the robust control problem (Wang et al., 2016; Zhong et al., 2013). Note that all of the above results are obtained for regulation design.

In system and control fields, it is often of great significance to track a desired trajectory with specific optimality performance. In particular, the trajectory tracking control problems have been studied under the adaptive critic framework (Kamalapurkar, Dinh, Bhasin, & Dixon, 2015; Modares & Lewis, 2014; Qu, Zhang, Feng, & Jiang, 2017; Vamvoudakis, Mojoodi, & Ferraz, 2017; Yang, Liu, Wei, & Wang, 2016; Zhang, Cui, Zhang, & Luo, 2011). The approximate optimal trajectory tracking problem of nonlinear systems was addressed in Kamalapurkar et al. (2015), Modares and Lewis (2014) and Zhang et al. (2011). The guaranteed cost tracking control method for a class of uncertain nonlinear systems was provided in Yang et al. (2016). The learning-based decentralized tracking problem for nonlinear large-scale interconnected systems was addressed in Qu et al. (2017). A novel event-triggered trajectory tracking controller of nonlinear systems was developed

☆ This work was supported in part by the National Natural Science Foundation of China under Grants U1501251, 61773373, 61533017, and 61233001, in part by Beijing Natural Science Foundation under Grant 4162065, and in part by the Early Career Development Award of SKLMCCS.

* Corresponding author at: The State Key Laboratory of Management and Control for Complex Systems, Institute of Automation, Chinese Academy of Sciences, Beijing 100190, China.

E-mail addresses: ding.wang@ia.ac.cn (D. Wang), derongliu@foxmail.com (D. Liu), yz@gdut.edu.cn (Y. Zhang), lihongyi2009@gmail.com (H. Li).

http://dx.doi.org/10.1016/j.neunet.2017.09.005
0893-6080/

in Vamvoudakis et al. (2017). Note that most of these results are derived for trajectory tracking of normal nonlinear systems without dynamical uncertainties.

As an important algorithm of reinforcement learning, policy iteration has been utilized frequently in various adaptive-critic-based optimization designs. However, there is an obvious difficulty, that is the choice of initial admissible control laws (Jiang & Jiang, 2014; Vamvoudakis & Lewis, 2010; Wang et al., 2016; Zhong et al., 2013). Besides, the adaptive-critic-based robust control approach is often applicable to regulation problem, rather than trajectory tracking (Fan & Yang, 2016; Jiang & Jiang, 2014; Wang et al., 2016; Zhong et al., 2013). To overcome these drawbacks, in this paper, we develop a novel neural-network-based robust tracking control method for nonlinear systems with matched uncertainties, where the initial stabilizing controller is not needed.

In what follows, the robust tracking control statement and the augmented system construction are provided in Section 2. Then, the main design method including the neural network control implementation, the uniform ultimate boundedness (UUB) property, and the tracking guarantee is analyzed in Section 3. The simulation verification and the concluding remark are given in Sections 4 and 5, respectively. Besides, we list the main notations used in the paper. $\mathbb{R}$ stands for the set of all real numbers. $\mathbb{R}^n$ is the Euclidean space of all n-dimensional real vectors. $\mathbb{R}^{n\times m}$ is the space of all $n\times m$ real matrices. $\|\cdot\|$ denotes the vector norm of a vector in $\mathbb{R}^n$ or the matrix norm of a matrix in $\mathbb{R}^{n\times m}$. I_n represents the $n\times n$ identity matrix and $0_{n\times m}$ stands for the $n\times m$ zero matrix. $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ calculate the maximal and minimal eigenvalues of a matrix, respectively. $\operatorname{diag}\{a_1, a_2, \ldots, a_n\}$ denotes the diagonal matrix composed of $a_1, a_2, \ldots, a_n$. Let Ω be a compact subset of $\mathbb{R}^n$ and $\mathscr{A}(\Omega)$ be the set of admissible control laws on Ω. The superscript "T" represents the transpose operation and $\nabla(\cdot) \triangleq \partial(\cdot)/\partial x$ denotes the gradient operator.

2. Robust tracking control problem statement

In this paper, let us consider a class of continuous-time nonlinear systems described by

$$\dot{x}(t) = f(x(t)) + g(x(t))[u(t) + d(x(t))], \tag{1}$$

where $x(t) \in \Omega \subset \mathbb{R}^n$ is the state vector and $u(t) \in \mathbb{R}^m$ is the control vector, $f(\cdot)$ and $g(\cdot)$ are known functions and are differentiable in their arguments with $f(0) = 0$, and $g(x)d(x)$ is the unknown perturbation with $d(0) = 0$. Here, we let $x(0) = x_0$ be the initial state and assume that the uncertain term $d(x)$ is bounded by a known function $\lambda_d(x)$, i.e., $\|d(x)\| \le \lambda_d(x)$ with $\lambda_d(0) = 0$.

For the purpose of tracking control, we denote $r(t) \in \mathbb{R}^n$ as the desired trajectory possessing the dynamics

$$\dot{r}(t) = \varphi(r(t)) \tag{2}$$

with the initial condition $r(0) = r_0$, where $\varphi(r(t))$ is a Lipschitz continuous function satisfying $\varphi(0) = 0$. Let the trajectory tracking error be $z(t) = x(t) - r(t)$ with the initial condition $z(0) = z_0 = x_0 - r_0$. Considering (1) and (2), we obtain the tracking error dynamics as

$$\begin{aligned}\dot{z}(t) = {} & f(z(t) + r(t)) - \varphi(r(t)) \\ & + g(z(t) + r(t))[u(t) + d(z(t) + r(t))].\end{aligned} \tag{3}$$

Next, we define an augmented state as the form $\xi(t) = [z^{\mathsf{T}}(t), r^{\mathsf{T}}(t)]^{\mathsf{T}} \in \mathbb{R}^{2n}$ with the initial condition $\xi(0) = \xi_0 = [z_0^{\mathsf{T}}, r_0^{\mathsf{T}}]^{\mathsf{T}}$. The augmented dynamics based on (2) and (3) can be formulated as

$$\dot{\xi}(t) = F(\xi(t)) + G(\xi(t))[u(t) + d(\xi(t))], \tag{4}$$

where $F(\cdot)$ and $G(\cdot)$ are new system matrices while $G(\cdot)d(\cdot)$ is the new uncertain term. In detail, they are written as

$$F(\xi(t)) = \begin{bmatrix} f(z(t) + r(t)) - \varphi(r(t)) \\ \varphi(r(t)) \end{bmatrix}, \tag{5a}$$

$$G(\xi(t)) = \begin{bmatrix} g(z(t) + r(t)) \\ 0_{n\times m} \end{bmatrix}, \tag{5b}$$

$$d(\xi(t)) = d(z(t) + r(t)). \tag{5c}$$

Note that the new uncertain term given above is still upper bounded since we find that

$$\|d(\xi)\| = \|d(x)\| \le \lambda_d(x) = \lambda_d(z + r) \triangleq \lambda_d(\xi). \tag{6}$$

For attaining the robust tracking purpose of system (1) to the reference trajectory (2), we construct the augmented dynamics (4) with the uncertainty $G(\xi)d(\xi)$ and aim to find a feedback control law $u(\xi)$ to ensure the closed-loop system to be stable. In what follows, we show that it can be transformed into designing the optimal controller of its nominal system

$$\dot{\xi}(t) = F(\xi(t)) + G(\xi(t))u(t). \tag{7}$$

We focus on the optimal feedback control design and want to find the control law $u(\xi)$ to minimize the cost function

$$J(\xi(t), u(t)) = \int_t^{\infty} \big\{\beta\lambda_d^2(\xi(\tau)) + U(\xi(\tau), u(\tau))\big\}\mathrm{d}\tau, \tag{8}$$

where β is a positive constant that will be designed and discussed later, $U(\xi, u)$ is the basic part of the utility function, $U(0, 0) = 0$, and $U(\xi, u) \ge 0$ for all ξ and u. Here, the basic utility function is chosen as the quadratic form $U(\xi, u) = \xi^{\mathsf{T}}\bar{Q}\xi + u^{\mathsf{T}}Ru$, where $\bar{Q} = \operatorname{diag}\{Q, 0_{n\times n}\}$, Q and R are positive definite matrices with $Q \in \mathbb{R}^{n\times n}$ and $R \in \mathbb{R}^{m\times m}$. The proposed cost function (8) reflects the uncertainty, regulation, and control terms simultaneously.

When studying optimal control problems, it is needed the designed feedback controller to be admissible (Vamvoudakis & Lewis, 2010; Wang et al., 2016). For any admissible control law $u \in \mathscr{A}(\Omega)$, if the associated cost function $J(\xi)$ is continuously differentiable, then its infinitesimal version is called the nonlinear Lyapunov equation

$$\begin{aligned}0 = {} & \beta\lambda_d^2(\xi) + U(\xi, u(\xi)) \\ & + (\nabla J(\xi))^{\mathsf{T}}[F(\xi) + G(\xi)u(\xi)]\end{aligned} \tag{9}$$

with $J(0) = 0$. Define the Hamiltonian as the form

$$\begin{aligned}H(\xi, u(\xi), \nabla J(\xi)) = {} & \beta\lambda_d^2(\xi) + U(\xi, u(\xi)) \\ & + (\nabla J(\xi))^{\mathsf{T}}[F(\xi) + G(\xi)u(\xi)].\end{aligned} \tag{10}$$

The optimal value of the cost function (8) is

$$J^*(\xi(t)) = \min_{u\in\mathscr{A}(\Omega)} J(\xi(t), u(t)), \tag{11}$$

which satisfies the HJB equation of the form

$$0 = \min_{u\in\mathscr{A}(\Omega)} H(\xi, u(\xi), \nabla J^*(\xi)). \tag{12}$$

The optimal feedback control law is derived by

$$u^*(\xi) = -\frac{1}{2}R^{-1}G^{\mathsf{T}}(\xi)\nabla J^*(\xi). \tag{13}$$

Taking the optimal control law (13) into (9), we can rewrite the HJB equation as

$$\begin{aligned}0 &= H(\xi, u^*(\xi), \nabla J^*(\xi)) \\ &= \beta\lambda_d^2(\xi) + U(\xi, u^*(\xi)) \\ &\quad + (\nabla J^*(\xi))^{\mathsf{T}}[F(\xi) + G(\xi)u^*(\xi)]\end{aligned} \tag{14}$$

with $J^*(0) = 0$. Next, we will cope with the neural network based robust tracking control design by using the adaptive critic framework.