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Variable structure controller design for Boolean networks

Liangjie Sun [a], Jianquan Lu [a,b,*], Yang Liu [c,a], Tingwen Huang [d], Fuad E. Alsaadi [e], Tasawar Hayat [b,f]

[a] School of Mathematics, Southeast University, Nanjing 210096, China
[b] Department of Mathematics, King Abdulaziz University, Jeddah 21589, Saudi Arabia
[c] College of Mathematics, Physics and Information Engineering, Zhejiang Normal University, Jinhua 321004, China
[d] Texas A&M University at Qatar, Doha 23874, Qatar
[e] Faculty of Engineering, King Abdulaziz University, Jeddah 21589, Saudi Arabia
[f] Department of Mathematics, Quaid-I-Azam University, Islamabad, Pakistan

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ABSTRACT

The paper investigates the variable structure control for stabilization of Boolean networks (BNs). The design of variable structure control consists of two steps: determine a switching condition and determine a control law. We first provide a method to choose states from the reaching mode. Using this method, we can guarantee that the number of nodes which should be controlled is minimized. According to the selected states, we determine the switching condition to guarantee that the time of global stabilization in the BN is the shortest. A control law is then determined to ensure that all selected states can enter into the sliding mode, such that any initial state can arrive in the steady-state mode. Some examples are provided to illustrate the theoretical results.

1. Introduction

Boolean network (BN) is the simplest logical dynamic system with binary state variables. Kauffman (1969) firstly proposed BNs as models of complex and nonlinear biological systems in 1969. The BN has been extensively used in describing, analyzing, and simulating the cellular networks and gene regulatory networks (Akutsu, Miyano, & Kuhara, 1999; Davidson, Rast, Oliveri, Ransick, Calestani, Yuh, Minokawa, Amore, Hinman, Arenas-Mena, et al., 2002; Huang, Li, Duan, & Starzyk, 2012; Shmulevich, Dougherty, Kim, & Zhang, 2002; Shmulevich, Dougherty, & Zhang, 2002). Moreover, it has been a powerful tool in capturing basic dynamic behavior and providing useful information for many real world systems. Recently, in Cheng and Qi (2010) and Cheng, Qi, and Li (2010), a new approach called the semi-tensor product (STP) of matrices, has been used successfully to express and analyze Boolean networks (BNs). Please refer to Lu, Li, Liu, and Li (2017) for a survey of STP. Based on the STP of matrices, it is easy to convert a BN with logical expression into an algebraic form and many fundamental results have been derived (Chen, Liang, & Wang, in press; Chen & Sun, 2013; Cheng, 2011; Cheng & Qi, 2009; Fornasini & Valcher, 2013; Laschov, Margaliot, & Even, 2013; Li & Lu, 2013; Li & Wang, 2015a, b; Li, Wang, & Xie, 2015; Li, Xie, & Wang, 2016; Li, Zhao, Weng, & Feng, in press; Liu, Chen, & Wu, 2014; Liu, Li, Lu, & Cao, in press; Liu, Sun, Lu, & Liang, 2016; Lu, Zhong, Ho, Tang, & Cao, 2016; Lu, Zhong, Huang, & Cao, 2016b; Lu, Zhong, Li, Ho, & Cao, 2015; Lu, Zhong, Tang, Huang, Cao, & Kurths, 2014; Luo, Wang, & Liu, 2014; Zhong, Lu, Huang, & Ho, 2017; Zhong, Lu, Liu, & Cao, 2014). The stabilization of BCNs is a basic but meaningful issue in control theory. So it has attracted great attention of system scientists. Many results about the stabilization of BCNs have been proposed in Cheng, Qi, Li, and Liu (2011), Li (2016), Liu, Cao, Sun, and Lu (2016), Li and Wang (2013) and Li, Yang, and Chu (2013, 2014).

Variable structure control is a kind of special nonlinear discontinuous control. It was firstly proposed by Emelyanov (1967) in 1950s. Furthermore, the theory of variable structure control was developed by Itkis (1976) and Utkin (1977, 1978). Variable structure control is an effective robust control strategy. Lots of researchers have been attracted since variable structure control is robust to uncertain parameters and external disturbances. Moreover, variable structure control can be applied in many practical systems (Chou & Cheng, 2003; De Battista & Mantz, 2004; Hsu, Chen, & Li, 2001; Jafarov, Alpaslan Parlakci, & Istefanopulos, 2005; Kim, Shin, & Chung, 2013). However, the variable structure control problem for BNs is still open and challenging, and to the best of our knowledge, there is no result on the construction of variable structure control for BNs as well.

Motivated by the above discussions, the objective of this paper is to design the variable structure control for stabilization of BNs.

* Corresponding author at: School of Mathematics, Southeast University, Nanjing 210096, China.
E-mail address: jqluma@seu.edu.cn (J. Lu).

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When the variable structure control is applied to the BN, the states response of the system can be separated into three modes including reaching, sliding, and steady-state modes. In other words, we expect that the BN's trajectory starting from any initial state converges to the steady-state mode. In this paper, if the state belongs to the sliding mode, it can arrive in the steady-state mode in finite time without any control. We just need to design the variable structure controller to control the states, which belong to the reaching mode, to enter into the sliding mode. The first problem we should solve in this paper is how to choose states from the reaching mode to guarantee that the number of nodes, which we should control, is minimized. Under the above precondition, we further determine the switching condition to ensure that the time of global stabilization in the BN is the shortest.

It consists of two steps for designing the variable structure controller of BNs in this paper. The first step is to determine a switching condition which can guarantee that any initial state can arrive in the sliding mode. We consider three kinds of steady-state modes (the steady-state mode is a fixed point, or the steady-state mode is a state in a limit cycle, or the steady-state mode is a transient state). For each situation, we separate all states into three modes named the reaching, sliding, and steady-state modes respectively. Then we present a method to choose states from the reaching mode. Using this method, we can guarantee that the number of nodes, which should be controlled is minimized. According to the selected states, we then determine the switching condition to guarantee that the time of the process of the BN global stabilization is the shortest. The second step is to determine a control law to guarantee that all selected states in the reaching mode, can enter into the sliding mode. The method of determining a control law is motivated by the algorithm in Li (2016). We only need to get part of the solution, and hence less computational complexity is required.

The rest is to be organized as the depiction below: Section 2 contains some preliminaries on STP. In Section 3, it consists of two steps for designing the variable structure controller of BN in this paper. The first step is to determine a switching condition and the second step is to determine a control law. Some examples are provided to illustrate the theoretical results. Conclusions are given in Section 4.

2. Preliminaries

For simplicity, we first give some notations. We denote $M_{m\times n}$ as the set of all $m \times n$ matrices. The delta set $\Delta_k := \{\delta_k^i | i = 1, 2, \ldots, k\}$, where δ_k^i is the ith column of identity matrix I_k with degree k. A matrix A is called a logical matrix if the columns set of A, denoted by $Col(A)$, satisfies $Col(A) \subset \Delta_m$, and let $Col_i(A)$ denote the ith column of matrix A. The set of all $m \times n$ logical matrices is denoted by $\mathcal{L}_{m\times n}$. Assuming $A = [\delta_m^{i_1}, \delta_m^{i_2}, \ldots, \delta_m^{i_n}] \in \mathcal{L}_{m\times n}$, we denote it as $A = \delta_m[i_1, i_2, \ldots, i_n]$ for simplicity. Let $\Omega_m = \{1, 2, \ldots, 2^m\}$. A logical domain $\mathcal{D}$, is defined by $\mathcal{D} = \{True = 1,\ False = 0\}$.

Definition 1 (Cheng et al., 2010). The semi-tensor product of two matrices $A \in M_{m\times n}$ and $B \in M_{p\times q}$ is defined as

$$A \ltimes B = (A \otimes I_{\alpha/n})(B \otimes I_{\alpha/p}),$$

where $\alpha = lcm(n, p)$ is the least common multiple of n and p, and $\otimes$ is the tensor (or Kronecker) product.

When $n = p$, STP is just the normal product. In this paper, we simply call STP "product" and omit the symbol "$\ltimes$" if no confusion raises.

Definition 2 (Cheng et al., 2010). An $mn \times mn$ matrix $W_{m,n}$ is called a swap matrix, if it is constructed in the following way: label its columns by $(11, 12, \ldots, 1n, \ldots, m1, m2, \ldots, mn)$ and its rows by $(11, 21, \ldots, m1, \ldots, 1n, 2n, \ldots, mn)$. Then its element in the position $((I, J), (i, j))$ is assigned as

$$w_{(I,J),(i,j)} = \delta_{i,j}^{I,J} = \begin{cases} 1, & I = i \text{ and } = j, \\ 0, & otherwise. \end{cases}$$

When $m = n$, we briefly denote $W_{[n]} = W_{[m,n]}$. Furthermore, for $X \in R^m$ and $Y \in R^n$, $W_{[m,n]} \ltimes X \ltimes Y = Y \ltimes X$, $W_{[n,m]} \ltimes Y \ltimes X = X \ltimes Y$.

Lemma 3 (Cheng et al., 2010). *Let $x = x_1x_2\cdots x_n$ with $x_i \in \Delta_2$, $(i = 1, 2, \ldots, n)$, then $x^2 = \Phi_n x$, where $\Phi_n = \delta_{2^{2n}}[1, 2^n+2, 2\cdot 2^n + 3, \ldots, (2^n-2)\cdot 2^n + 2^n - 1, 2^{2n}]$.*

To use matrix expression we identify each element in $\mathcal{D}$ with a vector as $True \sim \delta_2^1$ and $False \sim \delta_2^2$. Then the Boolean variable takes value from these two vectors, and $\mathcal{D} \sim \Delta_2$. Using STP of matrices, a logical function with n arguments $f : \mathcal{D}^n \to \mathcal{D}$ can be expressed in the algebraic form as follows:

Lemma 4 (Cheng et al., 2010). *Any logical function $f(x_1, \ldots, x_n)$ with logical arguments $x_1, \ldots, x_n \in \Delta_2$ can be expressed in a multi-linear form as*

$$f(x_1, \ldots, x_n) = M_f x_1 x_2 \cdots x_n,$$

where $M_f \in \mathcal{L}_{2\times 2^n}$ is unique, and called the structure matrix of f.

3. Main results

3.1. Problem formulation

A BN can be described as follows

$$\begin{cases} x_1(t+1) = f_1(x_1(t), \ldots, x_n(t)), \\ x_2(t+1) = f_2(x_1(t), \ldots, x_n(t)), \\ \vdots \\ x_n(t+1) = f_n(x_1(t), \ldots, x_n(t)), \end{cases} \tag{1}$$

where $t = 0, 1, 2, \ldots$ is the discrete time, $f_i : \mathcal{D}^n \to \mathcal{D}$ are logical functions, and $x_i \in \mathcal{D},\ i = 1, 2, \ldots, n$ are states of the BN.

In view of the vector expression of logic, let $x_i(t) \in \Delta$. Then, using Lemma 4, for each logical function $f_i,\ i = 1, 2, \ldots, n$, we can find its unique structure matrix M_i. Let $x(t) = \ltimes_{i=1}^n x_i(t)$. Then system (1) can be converted into an algebraic form as follows

$$\begin{cases} x_1(t+1) = M_1 x(t), \\ x_2(t+1) = M_2 x(t), \\ \vdots \\ x_n(t+1) = M_n x(t). \end{cases} \tag{2}$$

Multiplying the equations in (2) together yields

$$x(t+1) = Lx(t), \tag{3}$$

where $Col_i(L) = \ltimes_{j=1}^n Col_i(M_j)$, $i = 1, 2, \ldots, 2^n$.

When the variable structure control is applied, the response of such a system in general consists of three modes, namely, the reaching mode, sliding mode, and steady-state mode. The states response of the system can also be separated into the reaching, sliding, and steady-state modes. For convenience, let the set of all states, which are separated into the reaching mode, be RM. All states, which are separated into the sliding mode, belong to the set SM. Similarly, we denote the set SS by the set of steady-state mode.