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New results on global exponential dissipativity analysis of memristive inertial neural networks with distributed time-varying delays

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ABSTRACT

This paper is concerned with the global exponential dissipativity of memristive inertial neural networks with discrete and distributed time-varying delays. By constructing appropriate Lyapunov–Krasovskii functionals, some new sufficient conditions ensuring global exponential dissipativity of memristive inertial neural networks are derived. Moreover, the globally exponential attractive sets and positive invariant sets are also presented here. In addition, the new proposed results here complement and extend the earlier publications on conventional or memristive neural network dynamical systems. Finally, numerical simulations are given to illustrate the effectiveness of obtained results.

1. Introduction

During the past decades, various neural networks have been extensively investigated and successfully applied to associative memory, pattern recognition, fault diagnosis, automatic control engineering, robotic, signal processing (Cao, 2003; Chen & Zheng, 2010; Forti, Nistri, & Papini, 2005; Song & Wang, 2008; Xu, Lam, & Ho, 2006; Yu, Cao, & Wang, 2007; Zhang, Liu, Huang, & Wang, 2010). It has been proved that the membrane of a hair cell can be realized by equivalent circuits with an inductance in semicircular canals of certain animals (Angelaki & Correia, 1991; Ashmore & Attwell, 1985). Scholars also have proved that the charge or flux q of an electron element with inertial term can be inertial with the tendency to be unchanged (Wang, Helian, Wu, Lim, Guo, & Parker, 2010). So, bringing an inertial term into a neural system provides evident engineering and biological backgrounds. It has been proved that the dynamical behaviors would be more complex when the inertial item is introduced into neural networks (Babcock & Westervelt, 1986).

Compared to conventional neural networks with first order derivative of states, inertial neural networks are second order derivative of states, and little attention has been given to the inertial neural networks. Until now, several papers have been found about inertial neural networks. With the development and application of inertial neural networks, the studies of such nonlinear system are necessary and meaningful of both theoretical and potentially practical application.

In the real world, because the stability of neural networks is a prerequisite for the applications, considerable attention has been paid to the research on the problem of stability analysis, e.g., see Huang, Chan, Huang, and Cao (2007), Jiang and Teng (2004) and Song, Liang, and Wang (2009). On the other hand, time delays are frequently encountered in engineering (Jian & Wang, 2015), biological and economic systems (Zhang, Shen, & Chen, 2014; Zhang, Zhu, & Chen, 2011). Meanwhile, neural networks often have a spatial extent because of the presence of an amount of parallel pathways of varying axon size and lengths. Then, there may exist either a distribution of conduction velocities along these pathways or a distribution of propagation delays over a period of time in some cases, which may cause another type of time delays, namely, distributed time delays in neural networks. And these years appeared many works, e.g., see Jian and Wang (2015), Li, Lam, and Cheung (2012), Song and Wang (2008) and Wang, Liu, and Liu (2005).

The dissipativity is a generalization of Lyapunov stability, and the dissipative theory can offer an effective framework for stability analysis of nonlinear systems (Liao & Wang, 2003). Moreover, it also builds strong connections among physics, control engineering and system theory, and it has been applied to norm estimation, chaos, and robust control, see Song and Zhao (2005), Wang, She, Zhong, and Cheng (2016) and Wu, Shi, Su, and Chu (2013). Therefore, it is of both theoretical and practical importance to study the problem of dissipativity analysis of inertial neural networks with distributed time delays. Recently, the dissipativity analysis of inertial neural networks has studied in Muralisankar, Gopalakrishnan, and Balasubramaniam (2012), Qi, Li, and Huang (2015), Rakkiyappan, Kumari, Chandrasekar, and Krishnasamy (2016);

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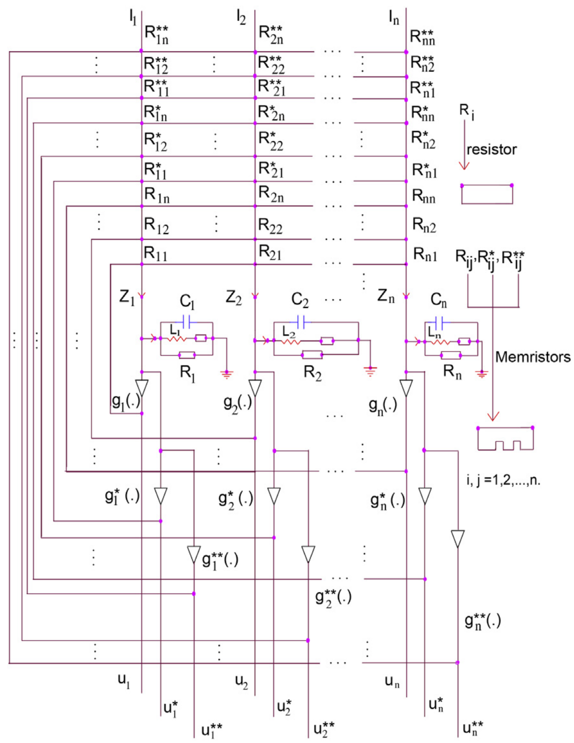

Fig. 1. Circuit of memristive inertial neural networks. R_{ij} is the memristor between the neuron activation function $g_j(\cdot)$ and $z_i(t)$, R^*_{ij} is the memristor between the neuron activation function $g^*_j(\cdot)$ and $z_i(t)$, R^{**}_{ij} is the memristor between the neuron activation function $g^{**}_j(\cdot)$ and $z_i(t)$, $I_i(t)$ is the external input, L_i is inductor, $z_i(t)$ is current of the inductor, R_i and C_i are the resistor and capacitor, u_i, u^*_i, u^{**}_i are the outputs, $i, j = 1, 2, \ldots, n$.

Rakkiyappan, Premalatha, Chandrasekar, and Cao (2016) and Tu, Cao, Alsaedi, and Alsaadi (2017).

It has been shown that memristor devices have many promising applications, one of which is to emulate synaptic behavior (Pershin & Di Ventra, 2010; Wang et al., 2012). And so, of course, we can replace resistors with memristors in the conventional circuit implementation of neural network to design a new model of neural networks to emulate the human brain, that is, the memristive neural networks. Memristive neural networks are a class of state-dependent nonlinear systems from a systems-theoretic point of view (Hu & Wang, 2010; Wu, Wang, Niu, & Wang, 2015; Wu, Wen, & Zeng, 2012; Wu & Zeng, 2014; Yang, Cao, & Yu, 2014; Zhang, Shen, Yin, & Sun, 2015). Such system family can reveal jumped, transient chaos of rich and complex nonlinear behaviors (Abdurahman & Jiang, 2016; Duan & Guo, 2016; Guo, Wang, & Yan, 2013; Guo, Yang, & Wang, 2016; Li & Cao, 2015; Zhang et al., 2014). In order to allow the memristors to be readily used in emerging technologies, the stability of such state-dependent nonlinear system family should be studied in the first position, as the above discussion, we know that the dissipative theory provides a nice tool for analyzing the stability of memristive neural networks.

Recently, the dissipativity analysis of memristive neural network was investigated in Guo et al. (2013) and Tu et al. (2017). However, on the dissipativity analysis of memristive inertial neural networks or conventional inertial neural networks with discrete and distributed time-varying delays, few results are found in the existing literatures. So, it is of great importance to fill this gap. Motivated by the above discussions and basing on the previous studies (Hu & Wang, 2010; Rakkiyappan, Kumari et al., 2016; Tu et al., 2017; Wu et al., 2012; Zhang et al., 2014), in this paper, we will derive several new criteria ensuring global exponential dissipativity of memristive inertial neural networks with discrete and distributed time-varying delays. The main contribution of this paper lies in the following aspects.

(1) Compared with the results on dissipativity of the neural networks with continuous right-hand side (Muralisankar et al., 2012; Qi et al., 2015; Wang et al., 2016; Wu et al., 2013), in this paper, we adopt nonsmooth analysis and dissipative theory to handle the global dissipativity of memristive inertial neural networks with discontinuous right-hand side, and our results of the dissipativity are more general and achieve a valuable improvement.

(2) The circuit implementation of memristive inertial neural networks with distributed time-varying delays is given out.

(3) The dissipativity analysis is extended to the memristive neural networks with inertial term and distributed time-varying delays, and the memristive inertial neural networks are second order derivative of states, which complement and extend the earlier publications.

(4) We consider the global dissipativity which is a generalization of Lyapunov stability. And the sufficient criteria in our paper can be directly derived from the parameters of the neural networks, and are very easily verified.

(5) The proposed method in this paper can be applied to the general nonlinear hybrid systems.

The organization of this paper is as follows. Model formulation and some preliminaries are introduced in Section 2. Several new criteria ensuring global exponential dissipativity of memristive inertial neural networks with discrete and distributed time-varying delays are derived in Section 3. Numerical simulations are given to demonstrate the effectiveness of the proposed results in Section 4. Finally, this paper ends with conclusions.

2. Preliminaries

Throughout this paper, solutions of all systems considered in the following are in Filippov's sense (Filippov, 1988). Let $\mathcal{N} = \{1, 2, \ldots, n\}$, $\mathbb{R}^n$ be the space of n-dimensional real column vectors. For any $\kappa = (\kappa_1, \kappa_2, \ldots, \kappa_n)^T \in \mathbb{R}^n$, the norms are defined by $\|\kappa\| = (\sum_{i=1}^{n}|\kappa_i|^p)^{1/p}$, where p is a positive integer and $p \geq 1$. $\mathrm{co}[\gamma^*, \gamma^{**}]$ denotes the convex hull of $\{\gamma^*, \gamma^{**}\}$. $\mathcal{A}_{ij} = \max\{|a^*_{ij}|, |a^{**}_{ij}|\}$, $\mathcal{B}_{ij} = \max\{|b^*_{ij}|, |b^{**}_{ij}|\}$, $\mathcal{C}_{ij} = \max\{|c^*_{ij}|, |c^{**}_{ij}|\}$. $\overline{d}_i = \max\{d^*_i, d^{**}_i\}$, $\underline{d}_i = \min\{d^*_i, d^{**}_i\}$, $\overline{a}_{ij} = \max\{a^*_{ij}, a^{**}_{ij}\}$, $\underline{a}_{ij} = \min\{a^*_{ij}, a^{**}_{ij}\}$, $\overline{b}_{ij} = \max\{b^*_{ij}, b^{**}_{ij}\}$, $\underline{b}_{ij} = \min\{b^*_{ij}, b^{**}_{ij}\}$, $\overline{c}_{ij} = \max\{c^*_{ij}, c^{**}_{ij}\}$, $\underline{c}_{ij} = \min\{c^*_{ij}, c^{**}_{ij}\}$. $I_i = \max_{t\geq 0}\{|I_i(t)|\}$, $\underline{\mu}_i = (\underline{d}_1 - \lambda_1, \underline{d}_2 - \lambda_2, \ldots, \underline{d}_n - \lambda_n)^T$, $\overline{\mu}_i = (\overline{d}_1 - \lambda_1, \overline{d}_2 - \lambda_2, \ldots, \overline{d}_n - \lambda_n)^T$, $\nu^+_i = \max\{|\underline{\mu}_i\lambda_i - \alpha_i|, |\overline{\mu}_i\lambda_i - \alpha_i|\}$. $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$.

In this paper, based on Kirchhoff's current law and from the circuit of memristive inertial neural networks as shown in Fig. 1, the equation of the ith subsystem can be described by the following equations:

$$
\begin{aligned}
\mathbf{L}_i\mathbf{C}_i\frac{\mathrm{d}^2 z_i(t)}{\mathrm{d}t^2} &= -z_i(t) - \mathbf{L}_i[\sum_{j=1}^{n}(\frac{1}{\mathbf{R}_{ij}} + \frac{1}{\mathbf{R}^*_{ij}} + \frac{1}{\mathbf{R}^{**}_{ij}})\delta_{ij} \\
&+ \frac{1}{\mathbf{R}_i}]\frac{\mathrm{d}z_i(t)}{\mathrm{d}t} + \sum_{j=1}^{n}\frac{\delta_{ij}}{\mathbf{R}_{ij}}g_j(\mathbf{L}_i\dot{z}_j(t)) + \sum_{j=1}^{n}\frac{\delta_{ij}}{\mathbf{R}^*_{ij}} \\
&\times g_j(\mathbf{L}_i\dot{z}_j(t-\tau_j(t))) + \sum_{j=1}^{n}\frac{\delta_{ij}}{\mathbf{R}^{**}_{ij}}\int_{t-\sigma_j(t)}^{t} g_j(\mathbf{L}_i\dot{z}_j(s))\mathrm{d}s \\
&+ I_i(t) \quad t \geq 0, i \in \mathcal{N},
\end{aligned}
$$

where $\delta_{ij} = 1$, if $i \neq j$ holds, otherwise, -1. $\mathbf{L}_i$ is inductance, $z_i(t)$ is current of the inductor, $\mathbf{R}_i$ and $\mathbf{C}_i$ are the resistor and capacitor, respectively, $\mathbf{R}_{ij}$ represents the memristor between the neuron