Neural Networks 75 (2016) 97–109

Contents lists available at ScienceDirect

Neural Networks

journal homepage: www.elsevier.com/locate/neunet

Existence and global exponential stability of periodic solution of memristor-based BAM neural networks with time-varying delays

Hongfei Li, Haijun Jiang*, Cheng Hu

College of Mathematics and System Sciences, Xinjiang University, Urumqi, 830046, Xinjiang, PR China

ARTICLE INFO

Article history:
Received 8 May 2015
Received in revised form 2 October 2015
Accepted 11 December 2015
Available online 18 December 2015

Keywords:
Memristor
Periodic solution
Boundedness
BAM neural networks
Global exponential stability

ABSTRACT

In this paper, we investigate a class of memristor-based BAM neural networks with time-varying delays. Under the framework of Filippov solutions, boundedness and ultimate boundedness of solutions of memristor-based BAM neural networks are guaranteed by Chain rule and inequalities technique. Moreover, a new method involving Yoshizawa-like theorem is favorably employed to acquire the existence of periodic solution. By applying the theory of set-valued maps and functional differential inclusions, an available Lyapunov functional and some new testable algebraic criteria are derived for ensuring the uniqueness and global exponential stability of periodic solution of memristor-based BAM neural networks. The obtained results expand and complement some previous work on memristor-based BAM neural networks. Finally, a numerical example is provided to show the applicability and effectiveness of our theoretical results.

1. Introduction

Professor Chua (1971) introduced memristor in 1971, and until 2008, a research team at the Hewlett–Packard labs announced that they had build a prototype of a solid-state and nanometer-size memristor (Strukov, Snider, Stewart, & Williams, 2008; Tour & He, 2008; Ventra, Pershin, & Chua, 2009). Memristor is a passive two-terminal electronic device described by nonlinear relationship links charge and flux, the resistance of a voltage-controlled memristor is uniquely determined by the time history of voltage across it and is indefinitely storable by the device once the controlling source is turned off (Raja & Mourad, 2010; Ventra et al., 2009). Analogous to the plasticity of biological synapse, memristor can change its memristance by the historic current through itself. From then on, memristor device has been the focus of recent research in the electrical and electronic engineering communities (Chua, 1971; Raja & Mourad, 2010; Shi, Duan, & Wang, 2015; Strukov et al., 2008; Tour & He, 2008; Ventra et al., 2009). In the brain-like neuronmorphic circuits (Raja & Mourad, 2010; Shi et al., 2015), memristor may be used as a non-volatile memory switch, it replaces the traditional circuit structures, which consist of transistors and capacitors, to serve as synapse to transmit information between neurons (Shi et al., 2015).

Owing to this important feature, we can replace resistor with memristors to build some new models of neural networks that emulates the human brain, and enable us to further study the dynamical behaviors for comprehending the function of human brain (Cao & Wan, 2014; Duan, Hu, Dong, & Mazumder, 2015; Hu, Feng, Duan, & Liu, 2015; Pershin, Fontaine, & Di Ventra, 2009). For example, we know that a Hopfield neural network model can be implemented in a circuit where the connection weights are implemented by resistors. Motivated by these facts, by using memristors instead of resistors, many authors have studied a new model, where the connection weights change according to its state, that is, it is a state-dependent switching neural network dynamical systems, which is said to be the memristor-based neural network (Guo, Wang, & Yan, 2013; Hu & Wang, 2010; Wu & Zeng, 2012). Moreover, the analysis of the memristor-based neural networks has been found useful to address a number of interesting engineering tasks, such as static friction, impacting machines, power circuits, switching in electronic circuits and many others, therefore the dynamical behaviors of memristor-based neural networks have received a great deal of attention in the previous works (Cai & Huang, 2014; Chen, Zeng, & Jiang, 2014; Duan & Huang, 2014; Guo et al., 2013; Hu & Wang, 2010; Kim, Du, & Sheridan, 2015; Li & Cao, 2015; Mathiyalagan, Park, & Sakthivel, 2015; Nie, Zheng, & Cao, 2015; Qin, Wang, & Xue, 2015; Wan & Cao, 2015; Wu & Zeng, 2012; Zhang, Shen, & Yin, 2013).

* Corresponding author.
E-mail address: jianghaijunxju@163.com (H. Jiang).

http://dx.doi.org/10.1016/j.neunet.2015.12.006
0893-6080/

As is well known, the property of periodic oscillatory solutions is very interesting and valuable. It has found applications in pattern recognition, associative memories, learning theory and so on (Halanay, 1966; Krasnosel'skii & Zabreiko, 1984; Li, Bohner, & Wang, 2015; Mawhin & Gaines, 1977; Yong & Zheng, 1995; Yoshizawa, 1963; Zhang, Li, & Huang, 2015). Meanwhile, the delays are actually encountered in practical implementation, due to the finite switching speed of the neuron amplifiers. It also knows that time delay is one of the main sources for causing instability and poor performances of networks. Therefore, it is very important to study the global stability and periodicity of memristor-based neural networks with time-varying delays. There have been many results on the existence and global stability of periodic solution of memristor-based neural networks with time-varying delays (Cai & Huang, 2014; Chen et al., 2014; Duan & Huang, 2014; Wan & Cao, 2015; Zhang et al., 2013). For example, Wan and Cao (2015) studied the periodicity and synchronization in coupled memristive neural networks with supremums via generalized halanay inequality. The periodic solution problem of memristor-based neural networks was firstly proposed via Mawhin-like coincidence theorem in Chen et al. (2014) and Duan and Huang (2014). Zhang et al. (2013) proposed memristor-based systems and some testable algebraic criteria were derived to achieve existence and stability of periodic solution. Cai and Huang (2014) employed the set-valued version of Krasnoselskii' fixed point theorem in a cone to derive the existence of the positive periodic solution of memristor-based BAM neural networks with delays. However, to the best of our knowledge, there are very few works on the existence and stability of periodic solution of the memristor-based BAM neural network with time-varying delays.

Motivated by the above discussions, in this paper, we will deal with the problem of existence and global exponential stability of periodic solution for memristor-based BAM neural networks with delays, the method used here involves Yoshizawa-like theorem, functional differential inclusions theory and inequality technique. The main results, which are new and complement previously known results.

This paper is organized as follows. In Section 2, the model description and preliminaries are given. In Section 3, the existence of Filippov periodic solution of memristor-based BAM neural networks with discontinuous right-hand sides is considered. In Section 4, some new testable algebraic criteria are derived for ensuring the uniqueness and global exponential stability of periodic solution of memristor-based BAM neural networks. In Section 5, the applicability and effectiveness of our theoretical results are shown by a numerical example.

Notations. For any column vectors $z = (z_1, z_2, \dots, z_n)^T \in \mathbb{R}^n$ and $y = (y_1, y_2, \dots, y_n)^T \in \mathbb{R}^n$, $\langle z, y\rangle = z^T y = \sum_{i=1}^{n} z_i y_i$ represents the scalar product of z, y, where the superscript T denotes the transpose operator. If $z \in \mathbb{R}^n$, we define the Euclidean norm $\|z\| = [\sum_{i=1}^{n} |z_i|^p]^{\frac{1}{p}}$, $p \ge 1$. Given a set $\mathbb{E} \subset \mathbb{R}^n$, $\mu(\mathbb{E})$ denotes the Lebesgue measure of set $\mathbb{E}$, $\overline{co}[\mathbb{E}]$ denotes the convex hull of $\mathbb{E}$. Furthermore, if $z \in \mathbb{R}^n$ and $\rho > 0$, $\mathcal{B}(z, \rho)$ is the ball of center z and radius ρ. Given the function $V(z)\colon \mathbb{R}^n \to \mathbb{R}$, $\nabla V(z)$ denotes the gradient of $V(z)$ and $\partial V(z)$ means Clarke's generalized gradient of $V(z)$. For a given continuous ω-periodic function $f(t)$ defined on $\mathbb{R}$, we define $f^u = \sup_{t\in[0,\,\omega]} |f(t)|$, $f^l = \inf_{t\in[0,\,\omega]} |f(t)|$.

2. Model formulation and preliminaries

In this paper, referring to some relevant works in Cai and Huang (2014), Chen et al. (2014), Duan and Huang (2014), Guo et al. (2013), Hu and Wang (2010), Kim et al. (2015), Li and Cao (2015) Mathiyalagan et al. (2015), Qin et al. (2015), Wan and Cao (2015), Wu and Zeng (2012), Zhang et al. (2013), which deal with the detailed construction of some general classes of memristor-based neural networks from the aspects of circuit analysis and memristor physical properties. We consider a general class of memristor-based bidirectional associative memory neural networks with time-varying delays described by the following equations:

$$\begin{cases} \dfrac{dx_i(t)}{dt} = -a_i(t)x_i(t) + \displaystyle\sum_{j=1}^{m} a_{ij}(x_i(t))f_j(y_j(t)) \\ \quad + \displaystyle\sum_{j=1}^{m} b_{ij}(x_i(t))f_j(y_j(t-\tau(t))) + I_i(t), \\ \dfrac{dy_j(t)}{dt} = -c_j(t)y_j(t) + \displaystyle\sum_{i=1}^{n} c_{ji}(y_j(t))g_i(x_i(t)) \\ \quad + \displaystyle\sum_{i=1}^{n} d_{ji}(y_j(t))g_i(x_i(t-\tau(t))) + J_j(t), \end{cases} \tag{1}$$

for $i = 1, 2, \dots, n,\ j = 1, 2, \dots, m$, where n, m correspond to the number of units in a neural network, $x_i(t)$ denotes the state variable associated with the ith neuron, $y_j(t)$ denotes the state variable associated with the jth neuron, $a_i(t) > 0$ ($c_j(t) > 0$) represents the rate with which the ith (jth) unit will reset its potential to the resting state in isolation when disconnected from the network and external inputs at time t, respectively, $f_j(y_j(t))$ and $g_i(x_i(t))$ denote the neuron activation functions, $\tau(t)$ corresponds to the transmission delay and satisfy $0 \le \tau(t) \le \tau$ ($\tau = \max_{0\le t\le\omega}\{\tau(t)\}$ is a positive constant). $I_i(t)$, $J_j(t)$ are continuous ω-periodic external input functions, $a_{ij}(x_i(t))$, $b_{ij}(x_i(t))$, $c_{ji}(y_j(t))$ and $d_{ji}(y_j(t))$ are memristive connection weights, respectively, which are defined as follows:

$$a_{ij}(x_i(t)) = \begin{cases} a_{ij}^{*}, & |x_i(t)| < T_i, \\ a_{ij}^{**}, & |x_i(t)| > T_i, \end{cases}$$

$$b_{ij}(x_i(t)) = \begin{cases} b_{ij}^{*}, & |x_i(t)| < T_i, \\ b_{ij}^{**}, & |x_i(t)| > T_i, \end{cases}$$

$$c_{ji}(y_j(t)) = \begin{cases} c_{ji}^{*}, & |y_j(t)| < \Upsilon_j, \\ c_{ji}^{**}, & |y_j(t)| > \Upsilon_j, \end{cases}$$

$$d_{ji}(y_j(t)) = \begin{cases} d_{ji}^{*}, & |y_j(t)| < \Upsilon_j, \\ d_{ji}^{**}, & |y_j(t)| > \Upsilon_j, \end{cases}$$

for $i = 1, 2, \dots, n,\ j = 1, 2, \dots, m$, where $a_{ij}(\pm T_i) = a_{ij}^{*}$ or a_{ij}^{**}, $b_{ij}(\pm T_i) = b_{ij}^{*}$ or b_{ij}^{**}, $c_{ji}(\pm\Upsilon_j) = c_{ji}^{*}$ or c_{ji}^{**}, $d_{ji}(\pm\Upsilon_j) = d_{ji}^{*}$ or d_{ji}^{**}, switching jumps $T_i > 0$, $\Upsilon_j > 0$, $a_{ij}^{*}, a_{ij}^{**}, b_{ij}^{*}, b_{ij}^{**}, c_{ji}^{*}, c_{ji}^{**}, d_{ji}^{*}, d_{ji}^{**}$ are all constants.

For convenience, we mainly apply the framework of Filippov solutions in discussing the solution of memristor-based BAM neural networks (1) with time-varying delays. Furthermore, we also present some basic concepts and definitions about the set-value map, differential inclusion and nonsmooth analysis, which will be used in the paper. Firstly, we introduce the following concepts and definitions.

Definition 1 (*Aubin & Cellina, 1984*). Suppose that to every point z of a set $\mathbb{E} \subset \mathbb{R}^n$, there corresponds a nonempty set $F(z) \subset \mathbb{R}^n$, then $z \to F(z)$ is called a set-valued map from $\mathbb{E} \to \mathbb{R}^n$. F is said to have a fixed point if there is $z \in \mathbb{E}$ such that $z \in F(z)$.

Definition 2 (*Aubin & Cellina, 1984*). A set-valued map F with nonempty values is said to be upper semi-continuous (USC) at $z_0 \in \mathbb{E} \subset \mathbb{R}^n$, if $\beta(F(z), F(z_0)) \to 0$ as $z \to z_0$ (i.e., for any open set $\mathbb{N}$ containing $F(z_0)$, there exists a neighborhood $\mathbb{M}$ of z_0 such that $F(\mathbb{M}) \subset \mathbb{N}$). $F(z)$ is said to have a closed (convex, compact) image if for each $z \in \mathbb{E}$, $F(z)$ is closed (convex, compact).