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New exponential synchronization criteria for time-varying delayed neural networks with discontinuous activations[☆]

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ABSTRACT

This paper investigates the problem of exponential synchronization of time-varying delayed neural networks with discontinuous neuron activations. Under the extended Filippov differential inclusion framework, by designing discontinuous state-feedback controller and using some analytic techniques, new testable algebraic criteria are obtained to realize two different kinds of global exponential synchronization of the drive–response system. Moreover, we give the estimated rate of exponential synchronization which depends on the delays and system parameters. The obtained results extend some previous works on synchronization of delayed neural networks not only with continuous activations but also with discontinuous activations. Finally, numerical examples are provided to show the correctness of our analysis via computer simulations. Our method and theoretical results have a leading significance in the design of synchronized neural network circuits involving discontinuous factors and time-varying delays.

1. Introduction

As far as we know, the non-Lipschitz or discontinuous neuron activations widely exist in many practical neural networks. Usually, the discontinuities of activations are caused by some interesting engineering tasks such as switching in electronic circuits, dry friction, systems oscillating under the effect of an earthquake and so on (see Cortés, 2008, Filippov, 1988, Forti & Nistri, 2003, Liu, Chen, Cao, & Lu, 2011 and Luo, 2009). Unfortunately, the additional difficulties will arise if discontinuities of activation are considered in the neural network dynamical systems. Actually, this kind of dynamical neuron system is usually described by the differential equation system possessing discontinuous right-hand side. It should be pointed out that many results in the classical theory of differential equation have been shown to be invalid since the given vector field is no longer continuous. In this case, the continuously differentiable solution is not guaranteed for the discontinuous neuron system. Moreover, it is necessary to reveal what changes will occur for different dynamic behaviors when discontinuous activations are introduced into the neural networks. In order to overcome these difficulties, Forti et al. first introduced the theory of differential inclusion given by Filippov to investigate the dynamical behaviors of neural networks with discontinuous activations (Forti & Nistri, 2003). Since then, neural networks with discontinuous activations have received a great deal of attention. Under the new framework named Filippov differential inclusion framework (Filippov, 1988), many excellent results on dynamical behaviors have been obtained for neural networks with discontinuous activations (Allegretto, Papini, & Forti, 2010; Cai, Huang, Guo, & Chen, 2012; Forti, Grazzini, Nistri, & Pancioni, 2006; Forti, Nistri, & Papini, 2005; Huang, Cai, Zhang, & Duan, 2013; Huang, Wang, & Zhou, 2009; Liu & Cao, 2009; Liu, Cao, & Yu, 2012; Liu et al., 2011; Lu & Chen, 2005, 2008; Papini & Taddei, 2005). However, most of existing papers are focused on the existence and convergence of equilibrium and periodic solution (or almost periodic solution) for neural network models with discontinuous activations. To the best of our knowledge, there is not much research concerning more complex dynamical behaviors such as chaos, bifurcation and synchronization for neuron systems with discontinuous activations.

On the other hand, the issues of chaos synchronization have been extensively studied for a rather long time since the pioneering work of Pecora and Carroll in 1990 (see Pecora & Carroll, 1990). It is worth mentioning that synchronization means the dynamics of nodes share the same time-spatial property and can be induced by coupling or by external forces. In fact, synchronization is a typical collective behavior which can be found in a wide variety of research fields such as biological systems, meteorology

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and secure communications (see Collins & Stewart, 1993, Duane, Webster, & Weiss, 1999, Liao & Huang, 1999 and Mirollo, Strogatz, & Williams, 1990). There are many types of synchronization including complete synchronization, anti-synchronization, phase synchronization, etc. Nowadays, chaos synchronization of neural networks has become a hot research topic owing to its theoretical significance (see, for example, Cao, Wang, & Sun, 2007, Hoppensteadt & Izhikevich, 2000, Lu, Ho, & Wang, 2009 and Yang, Cao, Long, & Rui, 2010). Recently, the interest of synchronization problem is shifting to the networks with discontinuous neuron activations despite the fact that the synchronization is not easy to be realized because of the discontinuous vector field. In Liu and Cao (2010), the complete synchronization was considered for the delayed neural networks with discontinuous activation functions via approximation approach. In Liu, Cao et al. (2012) and Liu et al. (2011), the quasi-synchronization criteria were obtained for discontinuous or switched networks. That is to say, the synchronization error can only be controlled within a small region around zero, but cannot approach zero with time. In Yang and Cao (2013), the authors investigated the exponential synchronization of delayed neural networks with discontinuous activations by constructing suitable Lyapunov functionals. Also, Liu et al. got some sufficient conditions on synchronization of linearly coupled dynamical neuron systems with non-Lipschitz right-hand sides (Liu, Lu, & Chen, 2012). But the synchronization criteria were expressed in integral inequalities and the discontinuous functions were weakened to be weak-QUAD or semi-QUAD. It should be noted that such synchronization criteria may be not easily verified in practice and there still lack new and efficacious methods for realizing synchronization control of discontinuous neural networks. Moreover, the new controller for synchronization should be designed. In addition, in many practical applications of neural networks, time delays between neuron signals are typical phenomena due to internal or external uncertainties. Because of the finite speed of signal propagation and the finite switch speed of neuron amplifiers, the time-delays in neurons are usually time variant and sometimes vary dramatically with time (Hou & Qian, 1998; Huang, Ho, & Lam, 2005). Therefore, it is necessary for us to investigate the synchronization problems for time-varying delayed dynamical neuron systems with discontinuous activations via the Filippov differential inclusion framework.

Notations: Let $\mathbb{R}$ be the set of real numbers and $\mathbb{R}^n$ denote the n-dimensional Euclidean space. Given the column vectors $x = (x_1, x_2, \ldots, x_n)^{\mathrm{T}} \in \mathbb{R}^n$ and $y = (y_1, y_2, \ldots, y_n)^{\mathrm{T}} \in \mathbb{R}^n$, where the superscript T denotes the transpose operator, $\langle x, y\rangle = x^{\mathrm{T}}y = \sum_{i=1}^{n} x_i y_i$ represents the scalar product of x, y, while $\|x\|$ denotes any vector norm in $\mathbb{R}^n$. Given a set $\mathbb{E} \subset \mathbb{R}^n$, by meas($\mathbb{E}$) we mean the Lebesgue measure of set $\mathbb{E}$ in $\mathbb{R}^n$ and $\overline{\mathrm{co}}[\mathbb{E}]$ denotes the closure of the convex hull of $\mathbb{E}$. If $z \in \mathbb{R}^n$ and $\delta > 0$, $\mathcal{B}(z, \delta) = \{z^* \in \mathbb{R}^n : \|z^* - z\| \le \delta\}$ denotes the ball of δ about z. Given the function $V : \mathbb{R}^n \to \mathbb{R}$, ∂V denotes Clarke's generalized gradient of V.

The remainder of this paper is outlined as follows. In Section 2, the model description and preliminaries including some necessary definitions and lemmas are stated. In Section 3, the main results and their rigorous proofs are given. Some new exponential synchronization criteria for time-varying delayed neural networks with discontinuous activations are proposed via introducing discontinuous state-feedback controller. In Section 4, two numerical examples are provided to illustrate the theoretical results. Finally, some conclusions are drawn in Section 5.

2. Model description and preliminaries

In this paper, we consider the time-varying delayed neural networks described by the following differential equations:

$$\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = -c_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t-\tau(t))) + I_i, \quad i = 1, 2, \ldots, n, \tag{1}$$

where $x_i(t)$ denotes the state variable of the potential of the ith neuron at time t; $c_i > 0$ denotes the self-inhibition with which the ith neuron will reset its potential to the resting state in isolations when disconnected from the network; a_{ij} represents the connection strength of jth neuron on the ith neuron; $f_j(\cdot)$ denotes the activation function of jth neuron; I_i is the external input to the ith neuron; $\tau(t)$ denotes the time-varying transmission delay at time t and is a continuous function satisfying

$0 \le \tau(t) \le \tau$ (here τ is a nonnegative constant).

Throughout this paper, the discontinuous neuron activations in (1) are assumed to satisfy the following properties:

(H1) For each $i = 1, 2, \ldots, n$, $f_i : \mathbb{R} \to \mathbb{R}$ is continuous except on a countable set of isolate points $\{\rho_k^i\}$, where there exist finite right and left limits, $f_i^+(\rho_k^i)$ and $f_i^-(\rho_k^i)$, respectively. Moreover, f_i has at most a finite number of discontinuities on any compact interval of $\mathbb{R}$.

(H2) For every $i = 1, 2, \ldots, n$, there exist nonnegative constants L_i and p_i such that

$$\sup_{\xi_i \in \overline{\mathrm{co}}[f_i(u)],\, \eta_i \in \overline{\mathrm{co}}[f_i(v)]} |\xi_i - \eta_i| \le L_i |u - v| + p_i, \quad \forall u, v \in \mathbb{R}, \tag{$*$}$$

where

$$\overline{\mathrm{co}}[f_i(\theta)] = \left[\min\{f_i^-(\theta), f_i^+(\theta)\},\ \max\{f_i^-(\theta), f_i^+(\theta)\}\right]$$

for $\theta \in \mathbb{R}$.

Remark 1. In general, the constant p_i in the condition (H2) should not equal to zero due to the discontinuity of the function f_i. Therefore, there exists essential difference between the condition (H2) and the Lipschitz condition in the previous literature. Especially, if the discontinuous function f_i satisfies the condition (H1) and is monotonically non-decreasing, then the following condition (H3) is satisfied.

(H3) For every $i = 1, 2, \ldots, n$, there exist nonnegative constants L_i and p_i such that

$$\sup_{\xi_i \in \overline{\mathrm{co}}[f_i(u)],\, \eta_i \in \overline{\mathrm{co}}[f_i(v)]} (u - v)(\xi_i - \eta_i) \le L_i (u - v)^2 + p_i |u - v|, \quad \forall u, v \in \mathbb{R},$$

where

$$\overline{\mathrm{co}}[f_i(\theta)] = \left[\min\{f_i^-(\theta), f_i^+(\theta)\}, \max\{f_i^-(\theta), f_i^+(\theta)\}\right]$$

for $\theta \in \mathbb{R}$.

Actually, if f_i satisfies the condition (H1) and is monotonically non-decreasing, then for $\forall \xi_i \in \overline{\mathrm{co}}[f_i(u)]$, $\eta_i \in \overline{\mathrm{co}}[f_i(v)]$, we have $(u - v)(\xi_i - \eta_i) \ge 0$ which implies $|u - v|\,|\xi_i - \eta_i| = (u - v)(\xi_i - \eta_i)$. Multiplying both sides of the inequality ($*$) by $|u - v|$, we obtain

$$\sup_{\xi_i \in \overline{\mathrm{co}}[f_i(u)],\, \eta_i \in \overline{\mathrm{co}}[f_i(v)]} |u - v|\,|\xi_i - \eta_i| \le L_i |u - v|^2 + p_i |u - v|, \quad \forall u, v \in \mathbb{R}.$$

That is to say, the condition (H3) holds. So the condition (H3) is a special case of (H2). For example, there are two classes of different situations illustrated in Fig. 1 when the discontinuous activation function $f_i(\theta)$ is discontinuous at $\theta = 0$ and satisfies (H2) and (H3), respectively. Here, we might as well take the two different cases of the discontinuous activation function $f_i(\theta)$ as follows:

$$\text{Case (a)}: \ f_i(\theta) = \begin{cases} \tanh(\theta) - 1, & \text{if } \theta \ge 0, \\ \tanh(\theta) + 1, & \text{if } \theta < 0. \end{cases}$$

$$\text{Case (b)}: \ f_i(\theta) = \begin{cases} \theta + 1, & \text{if } \theta \ge 0, \\ \theta - 1, & \text{if } \theta < 0. \end{cases}$$

Since neural network (1) is a delayed differential equation system possessing discontinuous right-hand side, the existence