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New synchronization criteria for memristor-based networks: Adaptive control and feedback control schemes ☆

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ABSTRACT

In this paper, we investigate synchronization for memristor-based neural networks with time-varying delay via an adaptive and feedback controller. Under the framework of Filippov's solution and differential inclusion theory, and by using the adaptive control technique and structuring a novel Lyapunov functional, an adaptive updated law was designed, and two synchronization criteria were derived for memristor-based neural networks with time-varying delay. By removing some of the basic literature assumptions, the derived synchronization criteria were found to be more general than those in existing literature. Finally, two simulation examples are provided to illustrate the effectiveness of the theoretical results.

1. Introduction

Memristors were proposed by Prof. Chua in his seminal paper Chua (1971). They share many properties and the same unit of measurement as resistors, but cannot be replaced by any of the other three circuit elements: resistor, capacitor, or inductor. As the fourth basic passive circuit element, they were not noticed by many researchers until the memristor prototype was manufactured by the Hewlett–Packard laboratory (Strukov, Snider, Stewart, & Williams, 2008; Tour & He, 2008). The main property of the memristor is that its memristance depends on the magnitude and polarity of the voltage and on how long the voltage has been applied. Hence, its memristance M can represent the functional relationship between charge and flux: $d\varphi = Mdq$, (see Fig. 1). Because of its memory function, the memristor has attracted increased attention. It can simulate the human brain quite realistically. It also has many potential applications, for example, it could increase the starting speed of a computer substantially and extend cell phone battery life by several months.

The mathematical model of memristor-based neural networks is a special case of a switched discontinuous system (Brown, 1994; Huang, Qu, & Li, 2005; Lou & Cui, 2008), whose switching rule depends on the network's state. However, this model has its own special features. The common method for dealing with the switched system (Hou, Zong, & Wu, 2011; Lian & Zhang, 2011; Zhang & Yu, 2009) is unsuitable for memristor-based neural networks. It is discontinuous on the right-hand side, and the synchronization study for a discontinuous right-hand side system is not easy. Recently, the dynamic behavior of memristor-based neural networks has become a popular topic (Wang, Li, Peng, Xiao, & Yang, 2014; Wu & Zeng, 2012, 2013; Zhang & Shen, 2013; Zhang, Shen, & Wang, 2013). Wu and Zeng (2012) studied closed-loop control problems of memristive systems, by designing optimal controllers. Some sufficient conditions in terms of linear matrix inequalities were obtained to ensure exponential stabilization of memristive cellular neural networks. By applying the drive–response concept, two different types of feedback controller were proposed to ensure exponential stability for the anti-synchronization error system in Wu and Zeng (2013). Chen, Zeng, and Jiang (2014) considered the model of fractional-order memristor-based neural networks (FMNN). They firstly proved the existence and uniqueness of its equilibrium point, then presented the sufficient criteria for global

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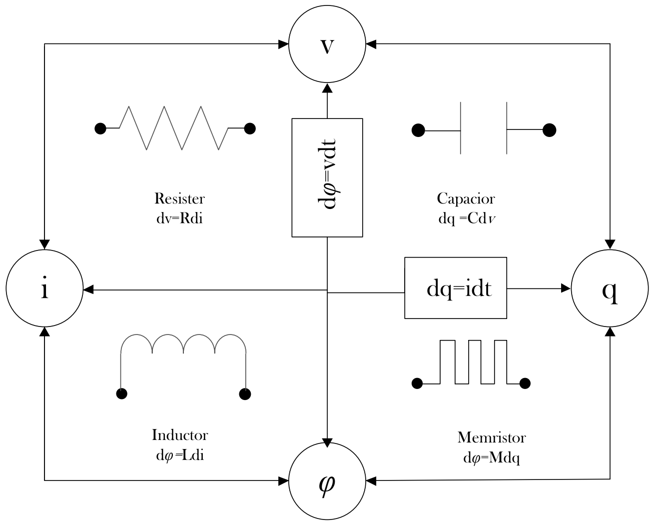

Fig. 1. Connection of four basic electrical elements (inspired by Strukov et al., 2008).

Mittag-Leffler stability and synchronization of the networks. Yang, Cao, and Yu (2014) discussed the problem of global exponential synchronization for a class of memristor-based Cohen–Grossberg neural networks with time-varying discrete delay and unbounded distributed delay. Through a nonlinear transformation, an alternative system of memristor-based Cohen–Grossberg neural networks was obtained. By designing a novel controller, corresponding synchronization criteria for memristor-based Cohen–Grossberg neural networks were given, the conditions established in the paper were improved upon, and the outcomes extended on those in existing papers.

The synchronization or anti-synchronization of memristor-based neural networks has received attention. In large-scale networks, they are unable to synchronize by themselves. Various effective control approaches and techniques have been proposed for synchronization. These include impulsive (Lu, Ho, Cao, & Kurths, 2011; Zhang & Sun, 2009), feedback (Cao & Wan, 2014; Rafikov & Balthazar, 2008), adaptive (Yang & Jiang, 2014; Zhou, Lu, & Lü, 2006), and intermittent (Huang, Li, Huang, & Han, 2013; Liu & Chen, 2011; Yang & Cao, 2009) control. The impulsive effects have been regarded as disturbances, the concept of average impulsive interval was used, and Lu et al. (2011) investigated the globally exponential synchronization of linearly coupled networks with impulsive disturbances. Authors have investigated drive–response fractional-order dynamic networks with uncertain parameters, by adopting an adaptive controller, which has a more general and simpler expression form. The adaptive laws of parameters were introduced by Yang and Jiang (2014). Adaptive controllers obtain effective results in actual applications. By designing suitable adaptive laws, adaptive controllers can adjust the coupling strength automatically. Furthermore, in the electronic implementation of memristor-based neural networks, time delays such as time-varying delays are inevitable because of the finite switching speed of the amplifiers, and they play an important role in the stability or synchronization of neural networks. They can result in network instability and should therefore be included (Cai & Huang, 2014; Cai, Huang, Guo, & Chen, 2012) in the mathematical model of memristor-based neural networks. To the best of our knowledge, no research exists on dealing with adaptive control for memristor-based neural networks with time-varying delay, despite its potential and practical importance.

Motivated by the aforementioned discussions, we deal with the synchronization of memristor-based neural networks with time delays using adaptive and feedback controllers, differential inclusion theory, and adaptive control techniques. By structuring novel Lyapunov functionals, an adaptive updated law is designed and new synchronization criteria for memristor-based neural network time-varying delays are proposed. Most previous work (Wang et al., 2014; Wu & Zeng, 2013; Zhang & Shen, 2013; Zhang et al., 2013) on the synchronization of memristor-based neural networks requires the basic assumption: $co\{\underline{a}_{ij}, \overline{a}_{ij}\}f_j(x_j(t)) - co\{\underline{a}_{ij}, \overline{a}_{ij}\}f_j(y_j(t)) \subseteq co\{\underline{a}_{ij}, \overline{a}_{ij}\}(f_j(x_j(t)) - f_j(y_j(t)))$. This assumption is not always derived. The main contributions in this paper can be summarized as follows: (1) a novel adaptive control law is designed to study the synchronization of memristor-based neural networks; (2) the time-varying delay is considered and a new mathematical model of memristor-based neural networks is established, which more closely approximates the actual model; and (3) basic assumptions in existing references are removed and new sufficient conditions are obtained to ensure that the memristor-based neural networks with time delay reach synchronization. This result is easy to verify and extends results from previous work.

In Section 2, the model formulation and some preliminaries are presented. In Section 3, adaptive synchronization criteria for memristor-based neural networks are obtained. In Section 4, synchronization criteria for memristor-based neural networks are derived by feedback control. Two numerical examples are given to demonstrate the validity of the proposed results in Section 5. Some conclusions are made in Section 6.

Notation: $\mathbb{R}$ denotes the set of real numbers, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $\mathbb{R}^{m\times n}$ denotes the set of all $m \times n$ real matrices. For $\tau > 0$, $C([-\tau, 0]; \mathbb{R}^n)$ denotes the family of continuous functions φ from $[-\tau, 0]$ to $\mathbb{R}^n$ with the norm $\|\varphi\| = \sup_{-\tau\le s\le 0} \max_{1\le i\le n} |\varphi_i(s)|$. The solutions of memristor-based networks are considered in Filippov's sense, and $[\cdot, \cdot]$ represents the interval. $co(Q)$ denotes the closure of the convex hull of Q. If not stated explicitly, matrices are assumed to have compatible dimensions for algebraic operations.

2. Model description and preliminaries

We consider the following memristor-based neural networks with time-varying delay:

$$\dot{x}_i(t) = -c_i(x_i(t)) + \sum_{j=1}^{n} a_{ij}(x_i(t))f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij}(x_i(t)) \\ g_j(x_j(t-\tau(t))) + I_i, \quad t \ge 0,\ i = 1, 2, \ldots, n, \tag{1}$$

where $x_i(t)$ is the voltage of the capacitor $\mathbf{C}_i$; $c_i(x_i(t))$ are appropriately behaved functions; $\tau(t)$ is the time-varying delay that satisfies differentiability and $0 \le \tau(t) \le \tau$, $\dot{\tau}(t) \le \sigma < 1$ where τ and σ are nonnegative constants; $f_j(\cdot)$ and $g_j(\cdot)$ are feedback functions; I_i is the external input; and

$$a_{ij}(x_i(t)) = \frac{\mathbf{W}_{ij}}{\mathbf{C}_i} \times \mathrm{sgin}_{ij}, \qquad b_{ij}(x_i(t)) = \frac{\mathbf{M}_{ij}}{\mathbf{C}_i} \times \mathrm{sgin}_{ij},$$

$$\mathrm{sgin}_{ij} = \begin{cases} 1, & i \ne j \\ -1, & i = j, \end{cases}$$

in which $\mathbf{W}_{ij}$ and $\mathbf{M}_{ij}$ denote the memductances of resistors $\mathbf{R}_{ij}$ and $\mathbf{F}_{ij}$, respectively. $\mathbf{R}_{ij}$ represents the resistors between the feedback function $f_i(x_i(t))$ and $x_i(t)$. $\mathbf{F}_{ij}$ represents the resistors between the feedback function $g_i(x_i(t-\tau(t)))$ and $x_i(t)$. According to the memristor features and the current–voltage characteristics, $a_{ij}(x_i(t))$ and $b_{ij}(x_i(t))$ are memristor-based connection weights that satisfy