Neural Networks 46 (2013) 183–189

Contents lists available at ScienceDirect

Neural Networks

journal homepage: www.elsevier.com/locate/neunet

Neural networks letter

Consistent stabilizability of switched Boolean networks ☆

Haitao Li *, Yuzhen Wang

School of Control Science and Engineering, Shandong University, Jinan 250061, PR China

ARTICLE INFO

Article history:
Received 27 November 2012
Received in revised form 7 April 2013
Accepted 24 May 2013

Keywords:
Switched Boolean network
Consistent stabilizability
State constraint
Semi-tensor product

ABSTRACT

This paper investigates the consistent stabilizability of switched Boolean networks (SBNs) by using the semi-tensor product method, and presents a number of new results. First, an algebraic expression of SBNs is obtained by the semi-tensor product, based on which the consistent stabilizability is then studied for SBNs and some necessary and sufficient conditions are presented for the design of free-form and state-feedback switching signals, respectively. Finally, the consistent stabilizability of SBNs with state constraints is considered and some necessary and sufficient conditions are proposed. The study of illustrative examples shows that the new results obtained in this paper are very effective in designing switching signals for the consistent stabilizability of SBNs.

1. Introduction

Network modelling has been widely applied to the analysis of cellular level biological systems, and many interesting results have been proposed for gene regulatory networks (El-Farra, Gani, & Christofides, 2005; Karlebach & Shamir, 2008; Marínez-Rodríguez, May, & Vargas, 2008; Wang, Lam, Wei, Fraser, & Liu, 2008), in which the gene regulatory network was modelled into Boolean networks (Kauffman, 1969), Bayesian networks (Marínez-Rodríguez et al., 2008) and differential equations (El-Farra et al., 2005; Wang et al., 2008), respectively. When using Boolean networks to model gene regulatory networks, gene expressions are quantized as 1 and 0 to represent active and inactive, respectively. Since Boolean networks are structurally simple, the study of Boolean networks has attracted a great deal of attention from scholars and many excellent results have occurred in a series of works (Akutsu, Hayashida, Ching, & Ng, 2007; Drossel, Mihaljev, & Greil, 2005; Kauffman, 1969; Kobayashi & Hiraishi, 2011). Recently, a novel matrix product, namely the semi-tensor product of matrices, has been proposed in Cheng, Qi, and Li (2011) and successfully applied to the analysis and control of Boolean networks. By this method, it is very convenient to convert a logical expression into an algebraic form, based on which many fundamental and landmark results have been presented for Boolean networks (Chen & Sun, 2013; Cheng, 2011; Cheng, Li, & Qi, 2010; Cheng & Qi, 2009; Cheng et al., 2011; Cheng & Zhao, 2011; Laschov & Margaliot, 2012; Li, 2012; Li & Chu, 2012; Li & Sun, 2011, 2012a, 2012b; Li & Wang, 2012a, 2012b; Li, Wang, & Liu, 2012; Zhao, Cheng, & Qi, 2010; Zhao, Li, & Cheng, 2011). In Cheng and Qi (2009), the controllability and observability of Boolean control networks were investigated, and a set of necessary and sufficient conditions were presented. The infinite horizon optimal control of logical control networks was considered in Zhao et al. (2011), and an optimal control was designed by the framework of game theory. The synchronization of two deterministic Boolean networks was studied in Li and Chu (2012), and some necessary and sufficient conditions were established based on the algebraic representation of logical dynamics. In Li and Sun (2011), the controllability of a μ-th order Boolean control network was investigated, and some necessary and sufficient conditions were provided by using a kind of input-state incidence matrix.

It is well worth pointing out that, while typical Boolean networks are described by purely discrete dynamics, the dynamics of biological networks in practice is often governed by different switching models (El-Farra et al., 2005). A practical example is the cell's growth and division in a eukaryotic cell, which are usually described as a sequence of four processes triggered by a set of conditions or events (Lewin, 2000). It is noted that, some other existing networks can also be converted to switched ones. For instance, a Boolean control network can be expressed as a Boolean switched system by encoding the control inputs as a switching signal (Laschov & Margaliot, 2012), and a deterministic asynchronous Boolean network can also be converted to a switched one by using the method given in Kobayashi and Hiraishi (2011). Thus, the logical switching phenomenon is often encountered in practice, and it is necessary for us to study the switched Boolean networks (SBNs). In the past three decades, due to the great importance of switched systems in both theoretical development and practical

☆ The Project is supported by the National Natural Science Foundation of China (G61074068, G61034007, G61174036), the Research Fund for the Taishan Scholar Project of Shandong Province, the Natural Science Foundation of Shandong Province (ZR2010FM013), the Scholarship Award for Excellent Doctoral Student granted by Ministry of Education, and the Graduate Independent Innovation Foundation of Shandong University (yzc10064).

* Corresponding author. Tel.: +86 15253130216; fax: +86 053188396412.
E-mail addresses: haitaoli09@gmail.com (H. Li), yzwang@sdu.edu.cn (Y. Wang).

0893-6080/$ – see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.neunet.2013.05.012

applications, the study of ordinary switched systems has drawn a great deal of attention, and a large number of results have been obtained for the systems' stability analysis and control designs (Liberzon & Morse, 1999; Sun, 2004, 2006; Sun & Ge, 2005; Trofino, Assmann, Scharlan, & Coutinho, 2009).

As one of the most important topics in the study of switched systems, the switching stabilizability was firstly proposed by Sun (2004) and then was studied in Sun (2006); Sun and Ge (2005); Trofino et al. (2009), which can be usually divided into two main classes (Sun, 2004; Sun & Ge, 2005): one is the so-called pointwise stabilizability, and the other is the consistent one. Obviously, the main advantage of the consistent stabilizability over the pointwise one is that it has strong robustness against the perturbation of initial states. In Li et al. (2012), we presented some necessary and sufficient conditions for the pointwise stabilizability of SBNs. However, to our best knowledge, there is no work available on the study of the consistent stabilizability for SBNs. In fact, it is a very challenging topic in that the variables of an SBN only take "1" and "0" and the existing methods for ordinary switched systems can hardly be applied to SBNs.

In this paper, using the semi-tensor product method, we investigate the consistent stabilizability of switched Boolean networks. Firstly, we convert the SBN into an algebraic form by the semi-tensor product. Secondly, we study the consistent stabilizability for the SBN based on the algebraic form, and present some necessary and sufficient conditions for the design of free-form and state-feedback switching signals, respectively. Finally, we consider the consistent stabilizability of the SBN with state constraints, and propose some necessary and sufficient conditions. The main contributions of this paper are as follows: (i) the semi-tensor product method is firstly applied to the consistent stabilizability of switched Boolean networks, and a new theoretical framework is established via this method. (ii) some necessary and sufficient conditions are given to design both free-form and state-feedback switching signals for the consistent stabilizability of the SBN without/with state constraints. These conditions are easily verified by using the MATLAB toolbox established by Cheng (http://lsc.amss.ac.cn/~dcheng/stp/STP.zip).

The rest of this work is structured as follows. Section 2 contains some preliminaries on the semi-tensor product of matrices and the algebraic expression of logical functions. In Section 3, we investigate the consistent stabilizability of SBNs without/with state constraints, and establish some necessary and sufficient conditions. Two illustrative examples are given in Section 4, which is followed by a brief conclusion in Section 5.

2. Preliminaries

In this section, we give some necessary preliminaries on the semi-tensor product of matrices and the algebraic expression of logical functions, which will be used in the sequel.

Definition 2.1 (*Cheng et al., 2011*)**.** The semi-tensor product of two matrices $A \in \mathbb{R}^{m\times n}$ and $B \in \mathbb{R}^{p\times q}$ is

$$A \ltimes B = (A \otimes I_{\frac{\alpha}{n}}) \left(B \otimes I_{\frac{\alpha}{p}}\right), \tag{2.1}$$

where $\alpha = lcm(n, p)$ is the least common multiple of n and p, I_n denotes the $n \times n$ identity matrix, and $\otimes$ is the Kronecker product.

Remark 2.2. It is noted that when $n = p$, the semi-tensor product of A and B becomes the conventional matrix product. Thus, the semi-tensor product of matrices is a generalization of the conventional matrix product. We can simply call it "product" and omit the symbol "$\ltimes$" if no confusion arises in the following.

Proposition 2.3 (Cheng et al., 2011)**.** *The semi-tensor product has the following properties:*

(i) *(Associative) Let* $A \in \mathbb{R}^{m\times n}, B \in \mathbb{R}^{p\times q}$ *and* $C \in \mathbb{R}^{r\times s}$. *Then* $(A \ltimes B) \ltimes C = A \ltimes (B \ltimes C)$.
(ii) *(Non-commutative but permutation equivalent)*
  (1) *Let* $X \in \mathbb{R}^{t\times 1}$ *be a column vector and* $A \in \mathbb{R}^{m\times n}$. *Then*
  $$X \ltimes A = (I_t \otimes A) \ltimes X. \tag{2.2}$$
  (2) *Let* $X \in \mathbb{R}^{m\times 1}$ *and* $Y \in \mathbb{R}^{n\times 1}$ *be two column vectors. Then*
  $$Y \ltimes X = W_{[m,n]} \ltimes X \ltimes Y, \tag{2.3}$$
  where $W_{[m,n]} \in \mathbb{R}^{mn\times mn}$ *is called the swap matrix (Cheng et al., 2011).*

The following notations will be used later.

(1) $\mathcal{D} := \{1, 0\}$, and $\mathcal{D}^n := \underbrace{\mathcal{D} \times \cdots \times \mathcal{D}}_{n}$.
(2) $\Delta_n := \{\delta_n^k \mid 1 \le k \le n\}$, where δ_n^k denotes the k-th column of the identity matrix I_n. For compactness, $\Delta := \Delta_2$.
(3) An $n \times t$ matrix M is called a logical matrix, if $M = [\delta_n^{i_1}\ \delta_n^{i_2} \cdots \delta_n^{i_t}]$. We express M briefly as $M = \delta_n[i_1\ i_2 \cdots i_t]$. Denote the set of $n \times t$ logical matrices by $\mathcal{L}_{n\times t}$.
(4) An $n \times t$ matrix $A = (a_{ij})$ is called a Boolean matrix, if $a_{ij} \in \mathcal{D}, \forall i = 1, \ldots, n, j = 1, \ldots, t$. Denote the set of $n \times t$ Boolean matrices by $\mathcal{B}_{n\times t}$. It is noted that, the Boolean matrix (unlike the logical matrix) may have more than one "1" in each column.
(5) $\mathrm{Col}_i(A)$ denotes the i-th column of the matrix A, and $\mathrm{Row}_i(A)$ denotes the i-th row of the matrix A.
(6) $Blk_i(A)$ denotes the i-th $n \times n$ block of an $n \times mn$ matrix A.

By identifying True $\sim 1 \sim \delta_2^1$ and False $\sim 0 \sim \delta_2^2$, we have $\Delta \sim \mathcal{D}$, where "$\sim$" denotes two different forms of the same object. In most places of this work, we use δ_2^1 and δ_2^2 to express logical variables and call them the vector form of logical variables.

The following lemma is fundamental for the matrix expression of logical functions.

Lemma 2.4 (Cheng et al., 2011)**.** *Let* $f(x_1, x_2, \ldots, x_s) : \mathcal{D}^s \mapsto \mathcal{D}$ *be a logical function. Then, there exists a unique matrix* $M_f \in \mathcal{L}_{2\times 2^s}$, *called the structural matrix of* f, *such that*

$$f(x_1, x_2, \ldots, x_s) = M_f \ltimes_{i=1}^s x_i, \quad x_i \in \Delta, \tag{2.4}$$

where $\ltimes_{i=1}^s x_i = x_1 \ltimes \cdots \ltimes x_s$.

For the conversion between $x_i = \delta_w^{\alpha_i}, i = 1, \ldots, \tau$ and the corresponding $\ltimes_{i=1}^\tau x_i = \delta_{w^\tau}^{\alpha}$, we have the following result.

Lemma 2.5 (Cheng et al., 2011)**.** *Assume that* $\ltimes_{i=1}^\tau x_i = \delta_{w^\tau}^{\alpha}$. *Then,* $x_i = \delta_w^{\alpha_i}, i = 1, \ldots, \tau$ *if and only if*

$$\alpha = \sum_{i=1}^{\tau-1} (\alpha_i - 1) w^{\tau-i} + \alpha_\tau.$$

Finally, we list the structural matrices for some basic logical operators which will be used later.

Negation ($\neg$): $M_n = \delta_2[2\ 1]$; Conjunction ($\wedge$): $M_c = \delta_2[1\ 2\ 2\ 2]$; Disjunction ($\vee$): $M_d = \delta_2[1\ 1\ 1\ 2]$; Conditional ($\rightarrow$): $M_i = \delta_2[1\ 2\ 1\ 1]$; Biconditional ($\leftrightarrow$): $M_e = \delta_2[1\ 2\ 2\ 1]$; Exclusive Or ($\bar{\vee}$): $M_p = \delta_2[2\ 1\ 1\ 2]$.

3. Main results

In this section, we investigate the consistent stabilizability of switched Boolean networks. We first give the problem formulation, and then establish some necessary and sufficient conditions for the case of the system without state constraints. Finally, we study the consistent stabilizability of the SBN with state constraints.