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Global anti-synchronization of a class of chaotic memristive neural networks with time-varying delays

Guodong Zhang, Yi Shen *, Leimin Wang

Department of Control Science and Engineering, Huazhong University of Science and Technology, Wuhan 430074, China
Key Laboratory of Image Processing and Intelligent Control of Education Ministry of China, Wuhan 430074, China

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ABSTRACT

This paper is concerned with the global exponential anti-synchronization of a class of chaotic memristive neural networks with time-varying delays. The dynamic analysis here employs results from the theory of differential equations with discontinuous right-hand side as introduced by Filippov. And by using differential inclusions theory, the Lyapunov functional method and the inequality technique, some new sufficient conditions ensuring exponential anti-synchronization of two chaotic delayed memristive neural networks are derived. The new proposed results here are very easy to verify and they also improve the earlier publications. Finally, a numerical example is given to illustrate the effectiveness of the new scheme.

1. Introduction

The concept of memristor (as a contraction of memory and resistor) was introduced and named by Prof. Chua in his seminal paper Chua (1971) in 1971. The existence of the memristor as the fourth ideal electrical circuit element (the other three fundamental circuit elements are the resistor, inductor and capacitor, respectively) was predicted in 1971 based on logical symmetry arguments, but it took scientists almost 40 years to invent a practical memristor device which was published by scientists in the literature (Strukov, Snider, Stewart, & Williams, 2008; Wang et al., 2012).

Two properties of the memristor attracted much attention. Firstly, its memory characteristic, and, secondly, its nanometer dimensions. The memory property and latching capability enable us to think about new methods for nano-computing, with the nanometer scale device providing a very high density and is less power hungry. From the previous work (Strukov et al., 2008; Wang et al., 2012), we know that the memristor exhibits features just as the neurons in the human brain have. Because of this feature, we can apply this device to build a new model of neural networks to emulate the human brain, and its potential applications are in next generation computers and powerful brain-like neural computers (Hu & Wang, 2010; Merrikh-Bayat & Shouraki, 2011; Pershin & Di Ventra, 2010).

As we know, neural networks can be implemented by VLSI circuits. For example, the Hopfield neural network model can be implemented in a circuit where the self-feedback connection weights and the connection weights are implemented by resistors. Suppose that we use memristors instead of resistors, then we can build a new model where the parameters change according to its state; i.e., it is a state-dependent switching neural network. Motivated by these facts, recently, the general memristor-based neural networks with time-varying delays in Hu and Wang (2010), Wu, Wen, and Zeng (2012), Wu and Zeng (2012), Wu and Zeng (2013), Zhang, Shen, and Sun (2012) and Zhang, Shen, Yin, and Sun (2013) have been proposed and studied as follows:

$$\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = -x_i(t) + \sum_{j=1}^{n} a_{ij}(x_i)f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij}(x_i) \times f_j(x_j(t-\tau_{ij}(t))), \quad t \ge 0,\ i = 1, 2, \ldots, n, \tag{1}$$

where

$$a_{ij}(x_i) = \begin{cases} a_{ij}^{*}, & |x_i(t)| < T_i, \\ a_{ij}^{**}, & |x_i(t)| > T_i, \end{cases}$$

$$b_{ij}(x_i) = \begin{cases} b_{ij}^{*}, & |x_i(t)| < T_i, \\ b_{ij}^{**}, & |x_i(t)| > T_i, \end{cases}$$

in which switching jumps $T_i > 0$, a_{ij}^{*}, a_{ij}^{**}, b_{ij}^{*}, b_{ij}^{**} are all constant numbers, and $\tau_{ij}(t)$ corresponds to the transmission delays and satisfies $0 \le \tau_{ij}(t) \le \tau$ (τ is a positive constant, $i, j = 1, 2, \ldots, n$).

* Corresponding author at: Department of Control Science and Engineering, Huazhong University of Science and Technology, Wuhan 430074, China. Tel.: +86 27 87543630; fax: +86 27 87543130.
E-mail addresses: zgdhbnu@163.com (G. Zhang), yishen64@163.com (Y. Shen).

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In the real world, stability and synchronization of chaotic systems are very important due to its potential applications in many different areas including secure communication, information science, biological systems, optics and so on, and over the years a lot of good results have appeared, see, e.g., Cao (2003), Cheng, Liao, Yan, and Hwang (2006), Chen, Wu, and Zhou (2008), Hu, Yu, Jiang, and Teng (2010), Liu, Wang, and Liu (2006). Shen and Wang (2009), Sheng and Yang (2008), Song (2009), Tang, Wang, Gao, Swift, and Kurths (2012), Tang, Gao, Zou, and Kurths (2012), Tang and Wong (2013), Wang, Liu, and Liu (2005) and Yu, Cao, and Wang (2007). And in recent years, the anti-synchronization control also has been successfully applied to many areas for image processing, information science and so on, see, e.g., Ahn (2009), Li and Zhou (2006), Meng and Wang (2007), Ren and Cao (2009), Wu and Zeng (2013), Yau (2008), Zhao and Zhang (2011) and Zhu and Cui (2007). In addition, by using anti-synchronization in communication systems, one may transmit digital signals by the transform between synchronization and anti-synchronization continuously, which will strengthen the security and secrecy. Therefore, the anti-synchronization problem is an important area of study.

The main contribution of this paper lies in the following aspects. First, the dynamic analysis here employs results from the theory of differential equations with discontinuous right-hand side as introduced by Filippov. Additionally, a linear feedback controller technique, which is totally different from the techniques employed in Hu and Wang (2010), Wu et al. (2012), Wu and Zeng (2012), Wu and Zeng (2013), Zhang et al. (2012), and Zhang et al. (2013), is used to study the stabilization and anti-synchronization of addressed neural networks with time-varying delays in this paper. Furthermore, some new criteria are derived to ensure stabilization and anti-synchronization of the neural networks. In addition, our main results are obtained based on p-norm and a large number of parameters are introduced in order to improve the generality, integrity and elegance of the results. Lastly, the new proposed results here are very easy to verify.

The organization of this paper is as follows. Some preliminaries are introduced in Section 2. In Section 3, some sufficient conditions for the exponential anti-synchronization are derived by constructing a suitable Lyapunov-like function. And then numerical simulations are given to demonstrate the effectiveness of the proposed approach in Section 4. Finally, this paper ends with a conclusion.

2. Preliminaries

Throughout this paper, solutions of all the systems considered in the following are intended in Filippov's sense (Filippov, 1988). And $[\cdot,\cdot]$ represents the interval. In Banach space of all continuous functions $\mathcal{C}([-\tau,0],R^n)$ equipped with the norm defined by $\|\psi\| = \sup_{-\tau\le t\le 0}[\sum_{i=1}^n |\psi_i(t)|^p]^{1/p}$, $p>1$, for all $\psi = (\psi_1(t), \psi_2(t),\dots,\psi_n(t)) \in \mathcal{C}([-\tau,0],R^n)$, $\mathrm{co}[\underline{\xi}_i, \overline{\xi}_i]$ denotes the convex hull. For a continuous function $k(t): R\to R$, $D^+k(t)$ is called the upper right dini derivative and defined as $D^+k(t) = \overline{\lim}_{h\to 0^+}\frac{1}{h}(k(t+h)-k(t))$. System (1) has the following form of initial condition: $x(s)=\phi(s)=(\phi_1(s),\phi_2(s),\dots,\phi_n(s))^T \in \mathcal{C}([-\tau,0],R^n)$. For convenience, now, we first introduce the following definitions about the set-valued map and differential inclusion (Aubin & Cellina, 1984; Clarke, Ledyaev, Stem, & Wolenski, 1998; Filippov, 1988).

Definition 1. Let $E\subset R^n$, $x\mapsto F(x)$ be called a set-valued map from $E\hookrightarrow R^n$, if to each point x of a set $E\subset R^n$, there corresponds a nonempty set $F(x)\subset R^n$.

Definition 2. A set-valued map F with nonempty values is said to be upper-semi-continuous at $x_0\in E\subset R^n$ if, for any open set N containing $F(x_0)$, there exists a neighborhood M of x_0 such that $F(M)\subset N$. $F(x)$ is said to have a closed (convex, compact) image if for each $x\in E$, $F(x)$ is closed (convex, compact).

Definition 3. For the system $\frac{dx}{dt} = g(x)$, $x\in R^n$, with discontinuous right-hand sides, a set-valued map is defined as

$$\Phi(x) = \bigcap_{\delta>0}\bigcap_{\mu(N)=0} \overline{\mathrm{co}}[g(B(x,\delta)\setminus N)],$$

where $\overline{\mathrm{co}}[E]$ is the closure of the convex hull of set E, $B(x,\delta)=\{y: \|y-x\|\le\delta\}$, and $\mu(N)$ is a Lebesgue measure of set N. A solution in Filippov's sense (Filippov, 1988) of the Cauchy problem for this system with initial condition $x(0)=x_0$ is an absolutely continuous function $x(t)$, $t\in[0,T]$, which satisfies $x(0)=x_0$ and the differential inclusion:

$$\frac{dx}{dt}\in\Phi(x), \quad \text{for a.e. } t\in[0,T].$$

By applying the theories of set-valued maps and differential inclusions above, the memristor-based neural network (1) can be written as the following differential inclusion:

$$\frac{dx_i(t)}{dt} \in -x_i(t) + \sum_{j=1}^n \mathrm{co}[\underline{a}_{ij},\overline{a}_{ij}]f_j(x_j(t)) + \sum_{j=1}^n \mathrm{co}[\underline{b}_{ij},\overline{b}_{ij}] \times f_j(x_j(t-\tau_{ij}(t))), \quad \text{for a.e. } t\ge 0,\ i=1,2,\dots,n, \tag{2a}$$

where

$$\underline{a}_{ij} = \min\{a^*_{ij}, a^{**}_{ij}\}, \qquad \overline{a}_{ij} = \max\{a^*_{ij}, a^{**}_{ij}\},$$
$$\underline{b}_{ij} = \min\{b^*_{ij}, b^{**}_{ij}\}, \qquad \overline{b}_{ij} = \max\{b^*_{ij}, b^{**}_{ij}\}.$$

And from Aubin and Cellina (1984), Clarke et al. (1998) and Filippov (1988), we know that the differential inclusion (2a) means that there exist $\widehat{a}_{ij}\in\mathrm{co}[\underline{a}_{ij},\overline{a}_{ij}]$, $\widehat{b}_{ij}\in\mathrm{co}[\underline{b}_{ij},\overline{b}_{ij}]$, such that

$$\frac{dx_i(t)}{dt} = -x_i(t) + \sum_{j=1}^n \widehat{a}_{ij}f_j(x_j(t)) + \sum_{j=1}^n \widehat{b}_{ij}f_j \times (x_j(t-\tau_{ij}(t))), \quad t\ge 0,\ i=1,2,\dots,n. \tag{2b}$$

Throughout this paper, we consider system (2a) or (2b) as the drive system and corresponding response system are as follows:

$$\frac{dy_i(t)}{dt} \in -y_i(t) + \sum_{j=1}^n \mathrm{co}[\underline{a}_{ij},\overline{a}_{ij}]f_j(y_j(t)) + \sum_{j=1}^n \mathrm{co}[\underline{b}_{ij},\overline{b}_{ij}] \times f_j(y_j(t-\tau_{ij}(t))) + u_i(t), \quad \text{for a.e. } t\ge 0, \tag{3a}$$

or

$$\frac{dy_i(t)}{dt} = -y_i(t) + \sum_{j=1}^n \widehat{a}_{ij}f_j(y_j(t)) + \sum_{j=1}^n \widehat{b}_{ij}f_j(y_j(t-\tau_{ij}(t))) + u_i(t), \quad t\ge 0,\ i=1,2,\dots,n, \tag{3b}$$

where $u_i(t)$, $i=1,2,\dots,n$ are the appropriate control input to obtain a certain control objective.

Now we do the following assumptions for the system (1):

(H1) For $\forall s_1,s_2\in R$, $s_1\neq s_2$, the neuron activation functions $f_i(i=1,2,\dots,n)$ are odd, bounded and satisfy the Lipschitz condition

$$|f_i(s_1)-f_i(s_2)|\le \rho_i|s_1-s_2|,$$

where $\rho_i>0$.