Neural Networks 35 (2012) 82–87

Contents lists available at SciVerse ScienceDirect

Neural Networks

journal homepage: www.elsevier.com/locate/neunet

α-stability and α-synchronization for fractional-order neural networks

Juan Yu, Cheng Hu, Haijun Jiang *

College of Mathematics and System Sciences, Xinjiang University, Urumqi, Xinjiang 830046, PR China

ARTICLE INFO

Article history:
Received 25 April 2012
Received in revised form 29 June 2012
Accepted 26 July 2012

Keywords:
Fractional-order
α-exponential stability
α-exponential synchronization

ABSTRACT

In this paper, a class of fractional-order neural networks is investigated. First, α-exponential stability is introduced as a new type of stability and some effective criteria are derived for such kind of stability of the addressed networks by handling a new fractional-order differential inequality. Based on the results, the existence and α-exponential stability of the equilibrium point are considered. Besides, the synchronization of fractional chaotic networks is also proposed. Finally, several examples with numerical simulations are given to show the effectiveness of the obtained results.

© 2012 Elsevier Ltd. All rights reserved.

1. Introduction

Fractional-order calculus is an area of mathematics that deals with derivatives and integrals from non-integer order. Although fractional calculus dates from the 17th century, fractional differential systems have recently been useful in diffusion, turbulence, electromagnetism, signal processing, and quantum evolution of complex systems. In the past few decades, the study of dynamics of fractional-order differential systems has attracted the interest of many researchers and many interesting and important results have been reported including the existence of chaos in factional-order differential systems (Deng & Li, 2005; Hartley, Lorenzo & Qammer, 1995; Lu & Chen, 2006; Wu, Li, & Chen, 2008), stability analysis (Delavari, Baleanu, & Sadati, 2012; Li, Chen, & Podlubny, 2009, 2010), and synchronization (Deng, Li, Wang, & Li, 2009; Ge & Ou, 2008; Odibat, 2010; Peng, Jiang, & Chen, 2008; Wu, Lu, & Shen, 2009).

Recently, fractional-order neural networks were presented and designed to distinguish the classical integer-order models (Arena, Caponetto, Fortuna, & Porto, 1998; Arena, Fortuna, & Porto, 2000; Boroomand & Menhaj, 2010; Petráš, 2006). In Petráš (2006), the authors pointed out that fractional derivatives provide an excellent tool for the description of memory and hereditary properties of various processes. Actually, fractional-order systems have infinite memory. Taking into account these facts, the incorporation of a memory term into a neural network model is an extremely important improvement. In literature (Lundstrom, Higgs, Spain, & Fairhall, 2008), the authors emphasized the utility of developing and studying fractional-order mathematical models of neural networks because fractional differentiation provides neurons with a fundamental and general computation ability that can contribute to efficient information processing, stimulus anticipation and frequency-independent phase shifts of oscillatory neuronal firing. It is also important to point out that fractional-order recurrent neural networks are expected to be very effective in applications such as parameter estimations due to the fact they are characterized by infinite memory. Therefore, it is necessary and interesting to study fractional-order neural networks both in theory and in applications.

Currently, some excellent results about fractional-order neural networks have been investigated, such as Kaslik and Sivasundaram (2011), Zhang, Qi, and Wang (2010), Zhou, Li, and Zhu (2008); Zhou, Lin, Zhang, and Li (2010) and Zhu, Zhou, and Zhang (2008). In Zhang et al. (2010), a fractional order three-dimensional Hopfield neural network was proposed and pointed out that chaotic behaviors can emerge in a fractional network. Besides, chaos control and synchronization for some simple fractional-order neural networks were proposed in Zhou et al. (2008, 2010) and Zhu et al. (2008) by mainly using Laplace transformation theory and numerical simulations. In Kaslik and Sivasundaram (2011, 2012), the local stability analysis for fractional-order neural networks of Hopfield type was discussed by applying the linear stability theory of fractional-order system.

To the best of our knowledge, there are few results on the global analysis of fractional-order neural networks. Motivated by this, several dynamical properties of fractional-order neural networks are proposed in this paper. The main contribution of this paper lies in the following aspects. First, a new fractional-order differential inequality is introduced, which leads to a new type of stability, that is, α-exponential stability. Different from traditional exponential stability, the convergent rate of the proposed exponential stability

* Corresponding author.
E-mail addresses: yujuanseesea@163.com (J. Yu), wacheng2003@163.com (C. Hu), jianghai@xju.edu.cn (H. Jiang).

0893-6080/$ – see front matter © 2012 Elsevier Ltd. All rights reserved.
doi:10.1016/j.neunet.2012.07.009

is related to the order of differentiation of the system. Besides, some novel criteria are derived to realize α-exponential stability by constructing a simple auxiliary function and applying analysis techniques. Based on the useful results, the existence and α-exponential stability of the equilibrium point are considered. Furthermore, the synchronization of fractional chaotic networks is also proposed. It is believed that all of those are new and useful for the design and application of fractional-order neural networks.

This paper is organized as follows. In Section 2, some definitions and lemmas of fractional differential and integral calculus are given. Some criteria for α-exponential stability of fractional-order neural networks are obtained in Section 3. Based on the criteria, the existence and α-exponential stability of the equilibrium point are also studied. In Section 4, the synchronization of fractional-order neural networks is proposed. The effectiveness and feasibility of the theoretical results are shown by some examples in Section 5.

2. Preliminaries

In this section, we will recall some definitions of fractional calculation and introduce some useful lemmas.

At present, there are three common definitions of fractional-order differential operator: Grunwald–Letnikov, Rieman–Liouville, and Caputo definitions. These three definitions are in general non-equivalent. However, the main advantage of the Caputo derivative is that it only requires initial conditions given in terms of integer-order derivatives, representing well-understood features of physical situations and thus making it more applicable to real world problems. For this, throughout this paper, we choose the Caputo fractional derivative.

Definition 1 (*Kilbas, Srivastava, & Trujillo, 2006*)**.** The fractional integral of order α for a function f is defined as

$$I^{\alpha}f(t)=\frac{1}{\Gamma(\alpha)}\int_{t_0}^{t}(t-\tau)^{\alpha-1}f(\tau)d\tau,$$

where $t \geq t_0$ and $\alpha > 0$.

Definition 2 (*Kilbas et al., 2006*)**.** Caputo's fractional derivative of order α for a function $f \in C^{n+1}([t_0,+\infty),R)$ is defined by

$$D^{\alpha}f(t)=\frac{1}{\Gamma(n-\alpha)}\int_{t_0}^{t}\frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha-n+1}}d\tau,$$

where $t > t_0$ and n is a positive integer such that $n-1<\alpha<n$.

Consider the initial value problem of the following fractional differential equation

$$\begin{cases} D^{\alpha}x(t)=f(t,x(t)), & t\in[0,+\infty),\\ x(0)=x_0, & x_0\in R^m \end{cases} \tag{1}$$

where $x=(x_1,x_2,\ldots,x_m)^T\in R^m, 0<\alpha<1, f:[0,+\infty)\times R^m\to R^m$ is continuous in t and locally Lipschitz in x.

Definition 3 (*Li et al., 2009, 2010, Delavari et al., 2012*)**.** The constant x^* is an equilibrium point of the Caputo fractional dynamic system (1) if and only if $f(t,x^*)=0$ for any $t\in[0,+\infty)$.

Definition 4. System (1) is said to be α-exponentially stable if there exist two positive constants M and λ such that any two solutions $x(t)$ and $y(t)$ of system (1) with different initial values denoted by x_0 and y_0, one has

$$\|x(t)-y(t)\|\leq M\|x_0-y_0\|\exp\left(-\lambda t^{\alpha}\right),\quad t\geq 0,$$

where $\|\cdot\|$ denotes the Euclidean norm.

Remark 1. Evidently, the exponentially convergent rate in Definition 4 is dependent on the order of differentiation of system. Especially, when α is small enough, α-exponential stability is equivalent to the common stability. When $\alpha=1$, system (1) is equivalent to an ordinal differential equation described by

$$\begin{cases} \dot{x}(t)=f(t,x(t)), & t\in[0,+\infty),\\ x(0)=x_0, & x_0\in R^m, \end{cases}$$

and α-exponential stability is reduced to the common exponential stability.

Lemma 1 (*Slotine & Li, 1991 Bellman–Gronwall Inequality*)**.** *Assume that function $y(t)$ satisfies*

$$y(t)\leq\int_0^t a(\tau)y(\tau)d\tau+b(t)$$

with $a(t)$ and $b(t)$ being known real functions. Then

$$y(t)\leq\int_0^t a(\tau)b(\tau)\exp\left[\int_{\tau}^{t}a(r)dr\right]d\tau+b(t).$$

If $b(t)$ is differentiable, then

$$y(t)\leq b(0)\exp\left[\int_0^t a(\tau)d\tau\right]+\int_0^t\dot{b}(\tau)\exp\left[\int_{\tau}^{t}a(r)dr\right]d\tau.$$

In particular, if $b(t)$ is a constant, we simply have

$$y(t)\leq b(0)\exp\left[\int_0^t a(\tau)d\tau\right].$$

The comparison theorem for fractional-order systems was introduced in Li et al. (2009, 2010) and Delavari et al. (2012), which is useful for later study.

Lemma 2 (*Li et al., 2009, 2010, Delavari et al., 2012*)**.** *If $D^{\alpha}x(t)\leq D^{\alpha}y(t)$ with $0<\alpha<1$ and $x(0)=y(0)$, then $x(t)\leq y(t)$.*

Lemma 3 (*Kilbas et al., 2006*)**.** *Let n is a positive integer such that $n-1<\alpha<n$. If $y(t)\in C^{n-1}[a,b]$, then*

$$I^{\alpha}D^{\alpha}y(t)=y(t)-\sum_{k=0}^{n-1}\frac{y^{(k)}(a)}{k!}(t-a)^k.$$

In particular, if $0<\alpha\leq 1$ and $y(t)\in C[a,b]$, then

$$I^{\alpha}D^{\alpha}y(t)=y(t)-y(a).$$

Lemma 4. *Let $V(t)$ be a continuous function on $[0,+\infty)$ and satisfies*

$$D^{\alpha}V(t)\leq\theta V(t),$$

where $0<\alpha<1$ and θ is a constant. Then

$$V(t)\leq V(0)\exp\left[\frac{\theta}{\Gamma(\alpha+1)}t^{\alpha}\right]. \tag{2}$$

Proof. According to Lemmas 2 and 3, we have

$$V(t)\leq V(0)+\frac{\theta}{\Gamma(\alpha)}\int_0^t(t-\tau)^{\alpha-1}V(\tau)d\tau,\quad t\geq 0.$$

Download English Version:

https://daneshyari.com/en/article/6863500

Download Persian Version:

https://daneshyari.com/article/6863500

Daneshyari.com