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Blind image deblurring by promoting group sparsity

Dong Gong [a], Rui Li [a], Yu Zhu [a], Haisen Li [a], Jinqiu Sun [b,*], Yanning Zhang [a]

[a] School of Computer Science and Engineering, Northwestern Polytechnical University, Xi'an, China
[b] School of Astronomy, Northwestern Polytechnical University, Xi'an, China

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ABSTRACT

Blind image deblurring aims to recover the sharp image from a blurred observation, which is an ill-posed inverse problem. Proper image priors for the unknown variables (*i.e.* latent sharp image and blur kernel) are crucial. Abundant previous methods have shown the effectiveness of the sparsity-based priors on both image gradients and the blur kernel. The correlation among the elements of the sparse variables is paid less attention, however. In this paper, we propose to handle the blind image deblurring problem by promoting group sparsity. The proposed group sparsity priors are based on the fact that the nonzero elements of natural image gradients and blur kernels tend to cluster in structured group pattern. Based on the proposed priors, we introduce proper algorithms to iteratively update latent image gradients and blur kernel, respectively. The proposed algorithms preserve the salient structures and smooth the minor components in image gradients and restrict the blur kernel in a domain of dynamic group sparse vector. To illustrate the reliability of the proposed algorithm, we conduct experiments to analyze the properties of the regularizers and the convergence property of the proposed algorithm. Experiments with both quantitative and visual comparison further prove the effectiveness of the proposed method.

1. Introduction

Image motion blur is a ubiquitous problem in image capturing, which is caused by various factors such as camera movement [1–3]. The extensive usage of the hand-held photography devices, *e.g.* cell-phones, has significantly increased the demand for image deblurring. To handle image blur, a conventional blur model assumes a spatially invariant blur kernel [1,4], and models the blurred image $\mathbf{y} \in \mathbb{R}^n$ as the convolution of a latent sharp image $\mathbf{x} \in \mathbb{R}^n$ and the blur kernel $\mathbf{k} \in \mathbb{R}^m$:

$$\mathbf{y} = \mathbf{k} * \mathbf{x} + \mathbf{n} \tag{1}$$

where $*$ denotes the convolution operator and $\mathbf{n} \in \mathbb{R}^n$ refers to an additive i.i.d. noise term sampled from an zero-mean Gaussian distribution [1,5][1]. The blur model can also be represented in matrix-vector form [4,5]:

$$\mathbf{y} = \mathbf{H}(\mathbf{k})\mathbf{x} + \mathbf{n} = \mathbf{A}(\mathbf{x})\mathbf{k} + \mathbf{n}, \tag{2}$$

where $\mathbf{H}(\mathbf{k}) \in \mathbb{R}^{n\times n}$ and $\mathbf{A}(\mathbf{x}) \in \mathbb{R}^{n\times m}$ are convolution matrics associated with $\mathbf{k}$ and $\mathbf{x}$, respectively.

Blind image deblurring seeks to recover the latent sharp image $\mathbf{x}$ and blur kernel $\mathbf{k}$ from the blurry image $\mathbf{y}$, which is a highly ill-posed problem. Solving the problem thus requires some powerful priors on both $\mathbf{x}$ and $\mathbf{k}$. For example, ℓ_1-norm [6,7] is often used on $\mathbf{k}$ to encourage the sparsity, the sparse gradient prior [1,2,7–10], dark channel prior [11] and some others [12] are often imposed on $\mathbf{x}$ to reduce the ill-posedness. Specifically, the sparsity-inducing priors are widely used for blind image deblurring [1,2,13] and some other image processing tasks [14,15] due to the usability. Previous methods have proven the effectiveness of sparsity-inducing priors on both $\mathbf{k}$ and (the gradients of) $\mathbf{x}$ [1,2,5,8,16] for blind image deblurring. They, however, are usually confined to promoting the sparsity by assuming that the components are *independent* for simplicity, which overlooks the inherent correlations among the elements in $\mathbf{x}$ or $\mathbf{k}$. In reality, the components of the blur kernel and image gradients are not only sparse but also correlated. In this paper, by focusing on the sparsity-inducing prior/regularization, we study how to improve the blind image deblurring performance by considering the correlation among the components in $\mathbf{x}$ or $\mathbf{k}$. Specifically, relying on promoting the group sparsity among the image gradients and the elements of the blur kernel, we propose a new effective alternating-optimization-based deblurring method. The main contributions of this paper can be summarized as:

- Unlike previous sparsity prior based kernel estimation methods that treat the variables independently, we propose to explore

* Corresponding author.
E-mail addresses: edgong01@gmail.com (D. Gong), lirui.david@gmail.com (R. Li), zhu_yu000@163.com (Y. Zhu), haisenli.nwpu@gmail.com (H. Li), sunjinqiu@nwpu.edu.cn (J. Sun), ynzhang@nwpu.edu.cn (Y. Zhang).

[1] **x, y, n** and **k** are represented in lexicographically ordered vectors.

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the correlation among the elements of the sparse variables and introduce a blur kernel estimation algorithm relying on promoting group sparsity of both image gradients and blur kernel.

- We propose a group sparsity-inducing regularizer on image gradients based on $\ell_{2,1}$-norm and a local group clustering pattern. Accordingly, an iteratively reweighting optimization method is proposed to solve for intermediate image gradients for kernel estimation. The proposed method updates the reweighting weights relying on the local group clustering pattern and is adaptive to the estimation error.
- Utilizing the group clustering property of natural blur kernel, we define a dynamic group sparsity [17] based feasible domain for the unknown blur kernel. An adaptive dynamic group sparsity projection based kernel updating method is also introduced to estimate blur kernel with group clustering property.

2. Related work

Existing blind image deblurring methods can be categorized into two main groups, maximum a posterior (MAP) methods [6,9,18] and variational Bayesian (VB) methods [1,2,4].

MAP methods minimize the negative log-posterior

$$-\log p(\mathbf{y}|\mathbf{x},\mathbf{k})p(\mathbf{x})p(\mathbf{k}),$$

where $p(\mathbf{y}|\mathbf{x},\mathbf{k})$ is the likelihood related to the imaging model (1), $p(\mathbf{x})$ and $p(\mathbf{k})$ denote the priors on $\mathbf{x}$ and $\mathbf{k}$, respectively. Minimizing the negative log-posterior can be achieved equivalently by addressing the problem:

$$\min_{\mathbf{k}\in\mathbb{K},\mathbf{x}\in\mathbb{X}} \frac{1}{2}\|\mathbf{y}-\mathbf{k}*\mathbf{x}\|_2^2+\lambda\Omega(\mathbf{x})+\gamma\Upsilon(\mathbf{k}), \quad (3)$$

where $\mathbb{X}$ and $\mathbb{K}$ denote the feasible domain of $\mathbf{x}$ and $\mathbf{k}$, respectively, $\Omega(\mathbf{x})$ and $\Upsilon(\mathbf{k})$ are the regularizers that reflect the priors, and λ and γ are regularization weights. Choosing appropriate priors on $\mathbf{x}$ and $\mathbf{k}$ or equivalent regularizers $\Omega(\mathbf{x})$ and $\Upsilon(\mathbf{k})$, is crucial for the performance.

Previous methods have shown that inducing sparsity of image gradients is helpful for kernel estimation [1,4,9]. To achieve this, some methods explicitly promote sparsity using some carefully designed regularizers. For example, Chan and Wong use the TV norm [19], *i.e.* an ℓ_1-norm on image gradients, as the regularizer on $\mathbf{x}$. Although this prior may favor a blurry image *in some certain conditions* [20], it can obtain high-quality results with a carefully designed optimization scheme [8]. Shan et al. [18] proposed a new regularizer by combining the ℓ_1-norm and a ring suppressing term. Some other regularizers are also proposed to suppress small structures for kernel estimation, including the ℓ_1/ℓ_2-norm regularization [7] and other reweighted norms [21]. Xu et al. [9] proposed an approximate ℓ_0-norm regularizer on image gradients. Zuo et al. [22] learn iteration-wise hyper-Laplacian prior on image gradients. Pan et al. [16] proposed to use ℓ_0-norm based regularizer on both image gradients and image pixels. In [10], a regularizer for promoting the sparsity of the image gradients and spectral analysis based regularizer on image blur kernel are used for blind image deblurring.

Rather than promoting sparsity relying on regularizers, some methods explicitly extract sparse salient structures for kernel estimation. For example, Joshi et al. [6,23] detect edges for kernel estimation. Cho and Lee [6] extract edges relying on a bilateral filter and a shock filter. Xu and Jia [24] select informative structures and remove small-scale structures based on the relative total variation. However, these edge prediction based methods usually rely on some manually designed operations. Gong et al. [5] proposed to estimate blur kernel by iteratively and gradually activating significant and beneficial non-zero gradients.

Although the existing gradient sparsity prior based MAP methods work well in many scenarios, the correlation among the sparse image gradient is less considered.

VB methods estimate the blur kernel by minimizing an upper bound of $-\log p(\mathbf{k}|\mathbf{y})$, which is the negative log-marginal distribution of $\mathbf{k}$, i.e., $p(\mathbf{k}|\mathbf{y})=\int_{\mathbf{x}} p(\mathbf{x},\mathbf{k}|\mathbf{y})d\mathbf{x}$ [1,2,4,20,25]. Wipf and Zhang [4] show that minimizing the upper bound using VB methods is equivalent to solving an MAP problem with a regularizer coupling $\mathbf{x}$ and $\mathbf{k}$.

3. Group sparsity priors for blind image deblurring

Given a blurred image, to achieve blind image deblurring, we first estimate the blur kernel and then recover the sharp image via *non-blind deconvolution* [26,27] using the estimated blur kernel. In practice, given an estimated blur kernel, we recover the sharp image using the sparsity prior based non-blind deconvolution method proposed in [26]. We will mainly focus on the blur kernel estimation problem.

In this section, we will introduce two group sparsity based priors for blur kernel estimation. The priors are specifically for natural image and blur kernel, respectively. For ease of modeling, we perform blur kernel estimation in the derivative domain of images [13]. With a slight abuse of notation, in the following, we will let $\mathbf{x}$ and $\mathbf{y}$ denote the lexicographically ordered image gradients of sharp and blurry images, respectively, *i.e.* $\mathbf{x}=\nabla\mathbf{x}$ and $\mathbf{y}=\nabla\mathbf{y}$, where ∇ denotes the operator to calculate image gradients[2].

Since convolution is commutative, the imaging model in (1) and blur kernel $\mathbf{k}$ are unaltered when $\mathbf{x}$ and $\mathbf{y}$ represent gradient image. We thus perform blur kernel estimation only on image gradients. Recall that the imaging model in (1) defines a likelihood $p(\mathbf{y}|\mathbf{x},\mathbf{k})$. By maximizing the likelihood, the blur kernel is to be estimated by solving

$$\min_{\mathbf{k},\mathbf{x}} \frac{1}{2}\|\mathbf{y}-\mathbf{x}*\mathbf{k}\|_2^2, \quad (4)$$

where the blur kernel $\mathbf{k}$ and sharp gradient image $\mathbf{x}$ are estimated simultaneously. Problem (4) is highly ill-posed, and thus requires priors to regularize the solution space of $\mathbf{x}$ and $\mathbf{k}$. We will then propose two novel regularization techniques for $\mathbf{x}$ and $\mathbf{k}$, respectively.

3.1. Group sparse prior on natural image gradients

Abundant previous studies have shown that the image gradients of sharp natural images tend to be sparse [1,2,8,13], *i.e.* many elements are zero (or nearly zero), while the values of a few elements are larger. They also prove the effectiveness of the sparse priors for kernel estimation [7,8]. Since the nonzero image gradients are usually related to some continuous contents, such as image edges and textures, the elements of image gradients tend to consistently cluster in groups by zero or nonzero element. To utilize the group clustering based correlations among the elements of the sparse image gradients, we propose to estimate blur kernel by promoting the group sparsity of the image gradients.

For a natural image, gradients in flat regions tend to be zero consistently while derivatives of edges or other high-frequency signal tend to be nonzero. Although the ℓ_1-norm based regularizer on image gradients may favor a trivial solution $\mathbf{x}=\mathbf{y}$ and $\mathbf{k}=\delta$ (identity kernel) *under some specific conditions* [20], it is usually powerful and performs well in practice [8]. The ℓ_1-norm based regularizer however treats the elements independently [28]. Based on the

[2] In this work, $\nabla=[\nabla_v^{\mathrm{T}},\nabla_h^{\mathrm{T}}]^T$, where $\nabla_v=[-1,1]^{\mathrm{T}}$ and $\nabla_h=[-1,1]$ denote the operators to calculate the image gradients on vertical and horizontal directions, respectively. Generally, other choices for ∇ are open, *e.g.* second order derivative operator.