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New results for uncertain switched neural networks with mixed delays using hybrid division method☆

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ABSTRACT

This paper addresses the problem of stability analysis for uncertain switched neural networks with mixed time-varying delays resorting to a novel delay division method. Firstly, based on the theory of arithmetic and geometric sequence, a hybrid division method is proposed to partition the delay interval into multiple subintervals with equal or unequal lengths. Secondly, a newly modified Lyapunov–Krasovskii function (LKF) including triple and quadruple integrals is established by considering the information of every subinterval. Thirdly, to deal with the derivative of LKF, the Wirtinger-based integral inequality and Peng-park's integral inequality are introduced. Finally, less conservative LMIs-based stability criteria are presented. Two numerical examples are provided to illustrate the feasibility and effectiveness of the proposed results.

1. Introduction

In the last few decades, a number of researches focus on the study of artificial neural networks for their existence in various fields, including pattern recognition, signal processing, associative memories and other scientific areas [1,2]. Therefore, the significance of studying neural networks (NNs) for many practical systems has been widely investigated [3–5,9,15]. Time delay, which is an unavoidable phenomenon, might lead to instability or other unfavourable situation that has bad influence on the application of NNs. Hence, the stability of delayed neural networks (DNNs) is of great importance and attracts extensive interests [7,8,11–14]. While, time delay in [11] was a constant and [12] concerned only the discrete time-varying delay with a zero lower bound. Actually, the existence of distributed delay caused by the structure of NNs need to be considered as well, in addition, when assuming time-varying delay $\tau(t)\in[\tau_m, \tau_M]$, τ_m is not restricted to be zero, which is more reasonable according to practical situation. Furthermore, parametric uncertainties are often inescapable resulted from parametric variations, modeling errors, and process uncertainties. Thus, there is great interest to create some effective methods and derive more general and applicable results for DNNs with uncertain parameters, discrete and distributed delays.

Maximum allowable delay bound (MADB) is a well known evaluation index to discuss the conservatism of the obtained stability criterion. Then, how to construct an appropriate LKF and how to deal with the derivative of LKF for obtaining further less conservative results become challenging tasks [35]. In view of the points mentioned above, various techniques are adopted to analyze the stability of DNNs. By augmented LKF, [16] reduced the conservativeness of stability criteria to a great extent. Meanwhile, it has been proved that, triple integral form LKF [5], delay-partition-dependent LKF [9,15,18] also have effect. In addition, numerous mathematical inequalities and techniques, for example, Jensen's inequality [18], reciprocally convex combination technique [19], free-matrix-based integral inequality [20], etc, are introduced to avoid unnecessary enlargement of the derivative of LKF.

Since delay-partitioning approach was first proposed in [18], much attention has been paid to stability analysis by means of this kind of method, for its effect on greatly improving the stability criteria and deriving less conservative results. In [9], the delay interval was divided into several equal subintervals. By taking a series of adjustable parameters into delay interval, the nonuniform partition method was proposed in [5,6]. A secondary partitioning method has been introduced in [21], which first separates

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the delay interval into two segments, ulteriorly divides these two segments later. It shows that, from [22], nonuniform partitioning method can result less conservatism than the results of uniform one. But the division with unequal segments in recent papers is presented by introducing numerous parameters which make the extra analytic and computational complexity. Therefore, there is great motivation to create a more effective division method.

On the other hand, switched systems have been studied extensively over the past decades, for its capacity in modeling practical systems and its potential applications [42,45–49]. Including a variety of subsystems as constituent parts, switched systems are governed by a switching rule to coordinate the switching. Recently, the stability problem of switched NNs has been investigated in [23–25]. By using quadratic convex combination method, [23] considered the stability of switched Hopfield NNs of neutral type. The analysis of globally asymptotical stability for switched NNs with mixed time delays was investigated in [24] based on a LMI approach. The authors in [25] discussed the problem of robust exponential stability for a class of uncertain switched NNs by exploring the mode-dependent properties of each subsystems. However, when revisiting the publications of switched NNs, we find that, on the one hand, there are few stability criteria obtained by using delay-partitioning method, which can make further improvement on the reduction of conservatism, on the other hand, the construction of LKF is so simple that the useful delay and activation function dependent items are arbitrary ignored.

Motivated by the above discussion, it is the first attempt to investigate the delay division method on the stability analysis for uncertain switched NNs with mixed time-varying delays. The main contributions of this paper are listed as follows.

(1) By introducing ρ, λ and q (three tunable parameters), a hybrid division method based on the theory of arithmetic and geometric sequence is first established to deal with the time varying delay switched NNs, which contains uniform or nonuniform partition technique as its special case. Using the proposed method, the delay interval $[\tau_m, \tau_M]$ is separated into multiple subintervals with the length subject to $(\rho + i\lambda)q^{\ell - i + 1}$.
(2) A new LKF with triple and quadruple integrals is constructed for stability analysis, which is related with the subintervals obtained by partitioning $[\tau_m, \tau_M]$. In addition, various inequalities are employed to bound integral terms in the derivative of LKF, respectively, which contributes to less conservative results.
(3) To facilitate expressing results, the augmented vectors e_i and $\eta(t)$ are given. Resorting to Finsler's lemma and Schur complement, the stability criteria are representing in LMIs-form, which can be solved easily by matlab LMI toolbox.

Notation: R^n denotes the n-dimensional Euclidean space, $R^{n\times m}$ represents the set of all $n \times m$ real matrices; I and 0 are, respectively, the identity matrix and zero matrix with appropriate dimensions; A^T and A^{-1} stand for the transpose and inverse of matrix A respectively; $diag\{\cdots\}$ symbolizes a diagonal matrix; the notation $P > 0$ ($P \geq 0$) means that P is real symmetric positive definite matrix (positive semidefinite matrix); $He\{X\}$ is defined as $He\{X\} = X^T + X$; "$*$" represents the elements below the main diagonal of a symmetric matrix; $C([-\sigma, 0], R^n)$ is the family of continuous functions ϕ from $[-\sigma, 0]$ to R^n. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. Problem description and preliminaries

This paper considers switched neural networks in the presence of discrete and distributed time-varying delays, with a set of individual neural networks as the subsystems and can be described as follows.

$$\begin{cases}\dot{x}(t) = -A_r(t)x(t) + B_r(t)g(x(t)) + C_r(t)g(x(t-\tau(t))) \\ \quad + D_r(t)\int_{t-d(t)}^{t} g(x(s))ds \\ x(t) = \phi(t), t \in [-\sigma, 0]\end{cases} \tag{1}$$

where r denotes a switching signal in the finite set $\mathcal{S} = \{1, 2, \ldots, N\}$, in other words, the matrices ($A_r(t)$, $B_r(t)$, $C_r(t)$, $D_r(t)$) take values in the finite set $\{(A_1(t), B_1(t), C_1(t), D_1(t)), (A_2(t), B_2(t), C_2(t), D_2(t)), \ldots, (A_N(t), B_N(t), C_N(t), D_N(t))\}$. Throughout this paper, the switching rule is assumed to be known priori to the receiver with available value. $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in R^n$ is the neuron state vector. $g(x(t)) = [g_1(x_1(t)), g_2(x_2(t)), \ldots, g_n(x_n(t))]^T \in R^n$ is the neuron activation function. $A_r(t)$, $B_r(t)$, $C_r(t)$, $D_r(t)$ are matrix functions with time-varying uncertainties, which are:

$$\begin{aligned}&A_r(t) = A_r + \Delta A_r(t)\ B_r(t) = B_r + \Delta B_r(t)\\ &C_r(t) = C_r + \Delta C_r(t)\ D_r(t) = D_r + \Delta D_r(t)\end{aligned} \tag{2}$$

where $A_r = diag\{a_{r1}, a_{r2}, \ldots, a_{rn}\} > 0$, B_r, C_r, D_r $(r = 1, 2, \ldots, N)$ are known real constant matrices. $\Delta A_r(t)$, $\Delta B_r(t)$, $\Delta C_r(t)$, $\Delta D_r(t)$ are unknown matrices representing time-varying parametric uncertainties in the system model. $\tau(t)$ is the time-varying delay satisfying $0 < \tau_m \leq \tau(t) \leq \tau_M$, for any $t \geq 0$, where τ_m and τ_M are constants. $d(t)$ is the distributed time-varying delay satisfying $0 \leq d(t) \leq d$, where d is a positive constant. $\sigma = \max\{\tau_M, d\}$, $\phi(t) \in C([-\sigma, 0], R^n)$ is the initial function.

In this paper, we denote $[0, \infty) \triangleq T_1 \bigcup T_2 \bigcup \cdots \bigcup T_N$, where T_k represents the set of running time of the kth subsystem, which means that when $t \in T_k$, the kth subsystem is activated.

An indicator function is defined as $\Pi_r(t)$, where

$$\Pi_r(t) = \begin{cases} 1\ , & t \in T_r \\ 0\ , & t \notin T_r \end{cases} \tag{3}$$

with $r = 1, 2, \ldots, N$. Then from (1), (2) and (3), the uncertain switched NNs with discrete and distributed time-varying delays can be described by

$$\begin{aligned}\dot{x}(t) = \sum_{r=1}^{N} \Pi_r(t)\Big(&-[A_r + \Delta A_r(t)]x(t) + [B_r + \Delta B_r(t)]g(x(t)) \\ &+ [C_r + \Delta C_r(t)]g(x(t-\tau(t))) \\ &+ [D_r + \Delta D_r(t)]\int_{t-d(t)}^{t} g(x(s))ds\Big)\end{aligned} \tag{4}$$

it follows that $\sum_{r=1}^{N} \Pi_r(t) = 1$, under any switching rule.

Then the following assumptions are given.

Assumption 1. The parameters $\Delta A_r(t)$, $\Delta B_r(t)$, $\Delta C_r(t)$, $\Delta D_r(t)$ $(r = 1, 2, \ldots, N)$ are time variant but norm bounded, which admit

$$\begin{aligned}&\begin{bmatrix}\Delta A_r(t) & \Delta B_r(t) & \Delta C_r(t) & \Delta D_r(t)\end{bmatrix}\\ &= NF(t)\begin{bmatrix}E_{r1} & E_{r2} & E_{r2} & E_{r4}\end{bmatrix}\end{aligned} \tag{5}$$

in which N, E_{ri} $(i = 1, 2, 3, 4)$ are known real constant matrices with appropriate dimensions, and $F(t)$ is an unknown matrix function satisfying $F^T(t)F(t) \leq I$.

Assumption 2. The neuron activation functions $g_i(\cdot)$ $(i = 1, 2, \ldots, n)$ are continuous, and satisfy the following condition: for any $s_1, s_2 \in R$, $s_1 \neq s_2$

$$k_i^- \leq \frac{g_i(s_1) - g_i(s_2)}{s_1 - s_2} \leq k_i^+ \tag{6}$$

where $g_i(0) = 0$, k_i^-, k_i^+ are known real constants. Throughout this paper, we denote $K^+ = diag\{k_1^+, k_2^+, \ldots, k_n^+\}$, $K^- = diag\{k_1^-, k_2^-, \ldots, k_n^-\}$, $K_1 = diag\{k_1^+k_1^-, k_2^+k_2^-, \ldots, k_n^+k_n^-\}$, $K_2 = diag\{(k_1^+ + k_1^-)/2, (k_2^+ + k_2^-)/2, \ldots, (k_n^+ + k_n^-)/2\}$, $K =$