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Three-step general discrete-time Zhang neural network design and application to time-variant matrix inversion

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ABSTRACT

In recent decades, neural networks methods have widely applied to many science and engineering fields. Zhang neural network (ZNN) as a special type of recurrent neural networks was proposed by Zhang et al, which has been applied to different time-variant problems solving. ZNNs are usually used to process the continuous-time signal in time-variant systems as continuous-time ZNN (CTZNN) models. Since digit devices and computers are widely applied in science and engineering, it is necessary to develop a general method to discretize CTZNN models to discrete-time ZNN (DTZNN) models. In previous work, Euler forward difference and two types of three-step Zhang et al. discretization (ZeaD) formulas were applied to discretize CTZNN models. In this paper, a three-step general ZeaD formula based on Taylor expansion is designed to approximate the first-order derivative of the target point, and discretize CTZNN models for time-variant matrix inversion. In comparison, the two types of three-step ZeaD formulas are the special cases of the proposed general ZeaD formula. For the situation of the time derivative of objective matrix unknown, two formulas of estimating the derivative are provided, and two other corresponding three-step general DTZNN models are proposed. Theoretical analyses present the stability and convergence of the three general DTZNN models for time-variant matrix inversion. The numerical experiment results substantiate the efficacy and superiority of the three proposed general DTZNN models for time-variant matrix inversion with the theoretical steady-state residual errors, comparing with those of Newton iteration and one-step DTZNN model. In addition, by comparing the numerical results of the two general DTZNN models for the situation of the time derivative of objective matrix unknown with the one of the derivative known, the steady-state residual errors of the formers are slightly bigger than the latter.

1. Introduction

In recent decades, with the remarkable feature such as distributed-storage and high-speed parallel-processing capability, convenience of hardware implementations and excellent performance in large-scale online applications, neural networks and neural-dynamics methods have widely applied to many science and engineering fields [1–8]. Due to the in-depth research in neural networks, the neural-dynamics approach based on recurrent neural networks is one of the important methods for solving time-variant problems [9,10]. In order to guarantee exponential convergence of the model, Zhang neural network (ZNN) as a special type of recurrent neural networks was proposed by Zhang et al., which has been applied to time-variant equation and inequality solving [11], optimization [12–14], and matrix inversion [15,16]. ZNNs are usually used to process the continuous-time signal in time-variant systems as continuous-time ZNN (CTZNN) models. Since digit devices and computers are widely applied in science and engineering, which receive and operate on signals in digital form, it is the main trend that most continuous-time methods will be discretized [17]. For the reasons, it is necessary to develop a general method to discretize CTZNN models to discrete-time ZNN (DTZNN) models for various real-world applications. For example, DTZNN models for time-variant matrix inverse are applied to robot manipulator, such as motion generation, tracking control and optimal control [18–20].

Neural dynamics of CTZNN models is usually presented as differential equations describing how the states change with respect to time. Numerical differentiation is the key to discretization of CTZNN models. Numerical differentiation, which describes methods for estimating the derivative of a mathematical function, is widely used to solve ordinary differential equations (ODEs) and partial differential equations (PDEs) in numerical analysis and engineering applications [21]. So far, a number of different meth-

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Table 1
Comparisons of numerical differentiation formulas in DTZNN models.

	Formula	n-step	General-form	Accuracy
This paper	Eq. (11)	3	Yes	High
[21,39]	$\dot{f}_k \doteq \frac{f_{k+1}-f_k}{\tau}$	1	No	Low
[11,12,40,42]	$\dot{f}_k \doteq \frac{2f_{k+1}-3f_k+2f_{k-1}-f_{k-2}}{2\tau}$	3	No	High
[12,41]	$\dot{f}_k \doteq \frac{6f_{k+1}-3f_k-2f_{k-1}-f_{k-2}}{10\tau}$	3	No	High

ods have been developed to construct useful difference formulas for first-order derivative approximation, such as undetermined coefficients [22,23], the finite difference [22,24–26], the regularization [27], the polynomial interpolation [28,29] and Richardson extrapolation [29]. In this paper, numerical differentiation is used to discretize CTZNN model. At present numerical differentiation formulas are mostly the finite difference approximation of derivatives, which cannot be used to the discretization of continuous-time models [24,30].

Matrix inversion is an essential topic of many solution in the different science and engineering fields, such as robotics control [31], machine learning [32], and optimization [33]. Generally speaking, the conventional methods for matrix inversion were designed for constant matrices [34–36]. However, many practical systems are time-variant, the corresponding matrices may also be time-variant. Thus, it is completely necessary to develop the corresponding methods for online time-variant matrix inversion. For time-variant matrix inversion, Getz and Marsden [37] proposed an explicit dynamic method which is convergent exponentially when the design parameter is sufficiently large and the initial value is sufficiently close to the theoretical one. Zhang and Ge [38] proposed a continuous-time ZNN (CTZNN) model for time-variant matrix inversion depicted in implicit dynamics, which is global exponential convergent. Subsequently an explicit CTZNN model was proposed by Guo and Zhang as a synthesis of the two above methods [39]. In order to implement ZNN in digit device, DTZNN models for time-variant matrix inversion [21,39–42] are proposed as the discretization results of CTZNN models by exploiting numerical differentiation techniques.

One-step DTZNN models are proposed by exploiting Euler forward difference formula to dicretize the CTZNN models [21,39]. In order to obtain better accuracy, a number of numerical differentiation formulas were proposed using Taylor-type numerical differentiation techniques by Zhang et al to discretize CTZNN models, which are called Zhang et al discretization (ZeaD) formulas. The corresponding discrete-time models are called multistep DTZNN models [11,12,40–42]. At present, most of multistep DTZNN models are three-step, of which the ZeaD formulas are presented as

$$\dot{f}_k \doteq \frac{2f_{k+1}-3f_k+2f_{k-1}-f_{k-2}}{2\tau}$$

or

$$\dot{f}_k \doteq \frac{6f_{k+1}-3f_k-2f_{k-1}-f_{k-2}}{10\tau},$$

where symbol $\doteq$ denotes the computational assignment operation. The details are shown and compared in Table 1.

Note that there perhaps are many ZeaD formulas of this kind. Thus, the problem is whether there exists a general ZeaD formula to generalize all the present three-step ZeaD formulas.

Remark 1. The general ZeaD formula is different from the general numerical differentiation formulas in references [22,24–26]. The methods in the references are effective to compute the derivatives of a function at specified points within its domain. However, the numerical differentiation formulas of them cannot be used to discretize continuous-time models for neglect of stability and convergence of the systems.

In this study, a three-step general ZeaD formula is designed to approximate the first-order derivative. Then, a three-step general DTZNN model is proposed for time-variant matrix inversion. For the situation of the derivative of the time-variant matrix $\dot{A}_k$ unknown, two other DTZNN models are proposed. By theoretical analyses and numerical experiments verification, the three proposed general DTZNN models are stable and convergent with order $O(\tau^3)$.

The rest of this paper is organized into five sections. Section 2 shows the problem formulation of time-variant matrix inversion and the formulation of CTZNN model, one-step and three-step general DTZNN models. In Section 3, a three-step general ZeaD formula is designed based on the Taylor expansion. Two formulas of estimation of $\dot{A}_k$ are established for the situation of $\dot{A}_k$ unknown, and three corresponding DTZNN models are proposed. The proposed DTZNN models for time-variant matrix inversion are investigated and analyzed in Section 4. Section 5 provides two illustrative numerical examples to substantiate the efficacy and superiority of the proposed three-step general DTZNN models for time-variant matrix inversion. Section 6 concludes this paper with final remarks. Before ending this introductory section, it is worth pointing out the main contributions of this paper as follows.

1) This paper provides a three-step general ZeaD formula design method based on bilinear transform. By considering the consistency and convergence, the ZeaD formula can be used not only to compute the derivatives of the target points, but also to discretize continuous-time models. However, the general numerical differentiation formula in references [22,24–26] cannot be applied to discretize continuous-time models. Besides, the three-step ZeaD formulas in up-to-date references [11,12,40,41] are just special cases of such a general formula.
2) In this paper, a three-step general DTZNN model for time-variant matrix inversion is proposed by exploiting the general ZeaD formula to discretize the CTZNN model. The stability and convergence of the DTZNN model are proved theoretically. Besides, for the situation of $\dot{A}_k$ unknown, two formulas of estimating $\dot{A}_k$ are provided, and two other three-step general DTZNN models are proposed, by which the actual application scope is further expanded.
3) The numerical experiment results substantiate the efficacy and superiority of the three proposed three-step general DTZNN models for time-variant matrix inversion with the error of order $O(\tau^3)$, comparing with those of Newton iteration and one-step DTZNN model.

2. Problem formulation, ZeaD formula and ZNN models

The problem formulation of time-variant matrix inversion, CTZNN and one-step DTZNN models are presented in this section. Then, general ZeaD formulas, DTZNN models and the objectives of them are presented.

2.1. Problem formulation

In this paper, the problem of time-variant matrix inversion is considered:

$$A(t)X(t) = I, \tag{1}$$

where, at any time instant t, $A(t) \in \mathbb{R}^{m\times m}$ is a nonsingular continuously differentiable time-variant coefficient matrix. $I \in \mathbb{R}^{m\times m}$ is the identity matrix, and $X(t) \in \mathbb{R}^{m\times m}$ is the time-variant unknown matrix to be obtained in our study.