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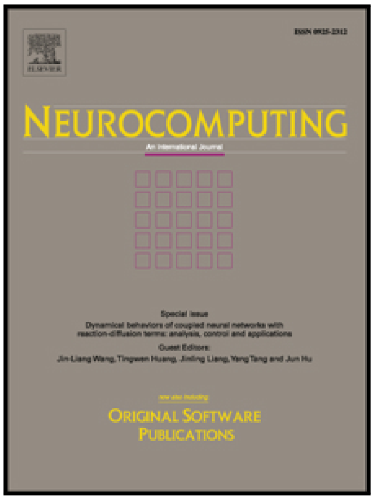

Gradient neural dynamics for solving matrix equations and their applications

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Gradient neural dynamics for solving matrix equations and their applications

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Abstract

We are concerned with the solution of the matrix equation $AXB = D$ in real time by means of the gradient based neural network (GNN) model, called $GNN(A, B, D)$. The convergence analysis shows that the result of global asymptotic convergence is determined by the choice of the initial state and coincides with the general solution of the matrix equation $AXB = D$. Several applications of the $GNN(A, B, D)$ model in online approximation of various inner and outer inverses with prescribed range and/or null space are considered. An appropriate adaptation of proposed models for finding an online solution of a set of linear equations $Ax = b$ is defined and investigated. The influence of various nonlinear activation functions on the convergence of $GNN(A, B, D)$ is investigated both theoretically as well as using computer-simulation results.

Keywords: Recurrent neural network; Moore-Penrose inverse; Drazin inverse; Dynamic equation; Matrix equation; Activation function.

AMS subject classifications: 68T05, 15A09, 65F20

1 Introduction

As usual, the set of all $m \times n$ real matrices is denoted by $\mathbb{R}^{m\times n}$. Similarly, the set of all $m \times n$ real matrices of rank r is denoted by $\mathbb{R}_r^{m\times n}$. Following the traditional notation, the column space, the null space and the rank of any matrix $A \in \mathbb{R}^{m\times n}$ are denoted by $\mathcal{R}(A)$, $\mathcal{N}(A)$ and $\mathrm{rank}(A)$, respectively. The index of A is denoted by $\mathrm{Ind}(A)$ it is defined as the smallest nonnegative integer k satisfying $\mathrm{rank}(A^k) = \mathrm{rank}(A^{k+1})$. Let $\sigma(A)$ be the spectrum of A and $s(A) = \mathrm{Re}\,(\sigma(A)) = \{\mathrm{Re}\,(\lambda) : \lambda \in \sigma(A)\}$; then $s(A) \geq 0$ denotes $\{\mathrm{Re}\,(\lambda) \geq 0 : \lambda \in \sigma(A)\}$. According to standard notation, $\|A\|_F := \sqrt{\mathrm{Tr}(A^{\mathrm{T}}A)}$ denotes the Frobenius norm of the matrix A, $\mathrm{Tr}(\cdot)$ denotes the trace of a matrix and I_n denotes the identity matrix of order n.

The Moore-Penrose inverse of a real matrix $A \in \mathbb{R}_r^{m\times n}$ is the unique matrix $X = A^{\dagger} \in \mathbb{R}_r^{n\times m}$ which satisfies the following Penrose equations:

$$(1)\quad AXA = A \qquad (2)\quad XAX = X \qquad (3)\quad (AX)^{\mathrm{T}} = AX \qquad (4)\quad (XA)^{\mathrm{T}} = XA.$$

A square matrix $X = A^{\mathrm{D}}$ is called a Drazin inverse of $A \in \mathbb{R}^{n\times n}$ if it fulfils the matrix equations

$$(1^k)\quad A^{l+1}X = A^l,\ l \geq \mathrm{Ind}(A), \quad (2)\quad XAX = X, \quad (5)\quad AX = XA. \tag{1.1}$$

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