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Exponential synchronization of Markovian jump complex dynamical networks with partially uncertain transition rates and stochastic disturbances☆

Yanan Wang, Yuechao Ma*, Aihong Chen

College of Science, Yanshan University, Qinhuangdao Hebei, 066004 PR China

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ABSTRACT

This paper studies the problem of exponential synchronization for a class of Markovian jump complex dynamical networks (MJCDNs) with stochastic disturbances and partially uncertain transition. By constructing the novel stochastic Lyapunov–Krasovskii function (LKF), and utilizing stochastic analysis, feedback pinning control technique and inequality techniques, some sufficient criteria are established in terms of linear matrix inequalities (LMIs) to guarantee the exponential synchronization of the MJCDNs with time delays and without it. Finally, according to a Markovian chain with partially uncertain transition rates, some numerical examples are given to demonstrate the effectiveness of the proposed results.

1. Introduction

Complex network is a kind of network which presents a high degree of complexity, its complexity is mainly manifested in three aspects: the first one is a large number of nodes; the second is the complex topology; and the third one is the complex dynamic behavior of each node. In addition to the power grid networks, the transportation networks, World Wide Web (WWW), the Internet and the interpersonal networks, in reality, the complex networks also include food webs, metabolic networks, etc. Because the complex networks system is dynamical, coupled with its complex dynamic behavior and control characteristics, the dynamic control and synchronization of complex dynamical networks system has become the focus of researchers [1–5].

During the past few years, Markovian jump system has been gaining increased research attention. The applications of the Markov jump systems can be sought in communication systems, network control systems, economic systems, modeling production systems, manufacturing systems and so on. In many practical engineering process, the random changes of the parameters such as changing coupling subsystem interconnections, the failure of the internal components or the influence of the external environment disturbance will lead to the change of the system parameters and the change of the topological structure, thus, the system can be switched between different model structures [6,7]. The results of previous studies were often derived from the assumption that the transition probabilities on the Markov jump systems are completely known [8–11], however, practically in general cases transition probabilities in the Markovian switching process are partly unknown or completely unknown. Therefore, it is necessary and significant to investigate more general jump systems with partially unknown transition probabilities. The problem of stability for Markovian jump systems with partly unknown transition probabilities has been discussed in [12–15]. In [16,17], the authors investigated the problems of synchronization for a class of Markovian jump networks with partly unknown transition probabilities.

In particular, the synchronization problem, as a significant and common phenomenon among various complex dynamical behaviors, has receive increasing attention and many significant advances have been studied extensively. Recently, with the further exploration of the complex network topology, especially since Watts and Strogatz introduced the small-world model to describe the more realistic networks [18], Barabsi and Albert found scale-free networks in 1999 [19], the dynamical characteristics of the network topology have attracted a lot of interest in variety of fields, including physics, biology and engineering [20,21]. There are

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* Corresponding author.
E-mail addresses: yuechaoma@ysu.edu.cn (Y. Ma), chenaihong@ysu.edu.cn (A. Chen).

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many different kinds of synchronization, such as exponential synchronization in [7,10], adaptive synchronization in [22,23], generalized synchronization in [24], lag synchronization in [25], etc. By using the periodically intermittent pinning control, several sufficient conditions are derived to ensure exponential synchronization for complex dynamical networks with stochastic perturbed in [8]. In [22], Yang and Cao investigated adaptive pinning synchronization of complex networks with non-delayed and delayed couplings and vector-form stochastic perturbations. In this paper, adaptive pinning controllers are designed to guarantee the synchronization of the complex networks even if partial states of the nodes are coupled. In [24], by utilizing the linear transformations theory, Yang and Chua presented the necessary and sufficient conditions to generalized synchronization between two chaotic systems. In addition, Wu and Lu [25] studied the projective lag synchronization phenomenon of the general complex dynamical networks with different nodes.

In this paper, our main aim is to derive the criteria to exponential synchronization for MJCDNs with partially uncertain transition rates and stochastic disturbances by the negative feedback pinning. The main motivation and contribution of this paper lie in three aspects: Firstly, we present a new Markov jump complex dynamical network model with delayed coupling and without it, by using the Lyapunov–Krasovskii functional method and the stochastic stability analysis theory, some novel sufficient conditions are obtained to guarantee exponential synchronization of the MJCDNs. Moreover, the complex network model of this paper is more general and practical, which is different from [26], the complex network model involves stochastic disturbances. Due to the stochastic perturbations are unavoidably affect the behavior of complex dynamical networks, the signals transmitted between nodes of Markov jump complex networks system or the subsystem are inevitably subject to stochastic disturbances from environment, which maybe affect the measurement of the transition probabilities and even break the stability of complex dynamical networks [27], Therefore, stochastic disturbances problems of MJCDNs are not be neglected. In this paper, based on the Itô formula and exponential synchronization theory, an feasible control method is present to guarantee the exponential synchronization of the MJCDNs with stochastic disturbances and partially uncertain transition. Finally, the control gain matrices of the feedback controllers have been derived in terms of LMIs which can be easily solved, and some numerical simulations are performed to verify the effectiveness and feasibility of the synchronization scheme.

The rest of this paper is organized as follows. In Section 2, a general model of Markov jump complex network with time-varying delays dynamical nodes and stochastic disturbances, and some preliminaries are proposed. Exponential synchronization criteria are derived and the negative feedback controllers are designed for the considered Markov jump complex networks in Section 3. Section 4 provides two numerical simulations to illustrate the theoretical results. Finally, some concluding remarks are drawn in Section 5.

Notation: Throughout this article, R^n and $R^{n\times m}$ denote the n-dimensional Euclidean space and the set of $n\times m$ real matrices,respectively. for symmetric matrices X and Y, the notation $X\geq Y(X\leq Y)$ means the matrix $X-Y$ is real positive definite (negative definite). The Kronecker product of matrices $A\in R^{m\times n}$ and $B\in R^{p\times q}$ are matrices in $R^{mp\times nq}$ and denote as $(A\otimes B)$. $\mathcal{F}$ is the σ-algebra of events, $\{\mathcal{F}_t\}_{t\geq 0}$ is increasing and right-continuous while $\mathcal{F}_0$ contains all $\mathcal{P}$-null sets, $\mathcal{P}$ is the probability measure defined on $\mathcal{F}$. I_N is the N-dimensional identity matrix, A^{T} denotes the transpose of matrix A, A^{-1} denotes the inverse of A. $\|\cdot\|$ stands for the 2-norm and $\|x\|=\sqrt{x^{\mathrm{T}}x}$, $diag\{\cdots\}$ represents a block-diagonal matrix, the asterisk $*$ is used to represent the symmetric term in a matrix.

2. Models and preliminaries

Let $\{r(t),\ t\geq 0\}$ be a right-continuous Markovian chain on a complete probability space $\left(\Omega,\ \mathcal{F},\ \{\mathcal{F}_t\}_{t\geq 0},\ \mathcal{P}\right)$ taking values in the finite state space $S=\{1,\ 2,\ \ldots,\ s\}$, with a generator $\Pi=(\pi_{rp})_{s\times s}$, $r,p\in S$ given by

$$p\{r(t+\Delta)|r(t)=r\}=\begin{cases}\pi_{rp}\Delta+o(\Delta), & r\neq p,\\ 1+\pi_{rp}\Delta+o(\Delta), & r=p.\end{cases} \tag{1}$$

Here $\Delta>0$ and $\lim_{\Delta\to 0}\frac{o(\Delta)}{\Delta}=0$, $\pi_{rp}=0$ is the transition rate from t to p at time $t+\Delta$, if $r\neq p$, while $\pi_{rr}=-\sum_{p=1,p\neq r}^{s}\pi_{rp}$.

Consider the following Markovian jump complex delayed network consisting of N identical nodes with diffusively couplings, in which each node is an n-dimensional dynamical system:

$$\begin{cases}\dot{x}_k(t)=-C(r(t))x_k(t)+A(r(t))f(x_k(t))+B(r(t))g(x_k(t-\tau(t)))\\ \qquad +c\sum_{l=1}^{N}G_{kl}(r(t))\Gamma x_l(t-\tau(t)),\quad k=1,2,\ldots,N,\\ x_k(t)=\xi_k(t)\in L_{\mathcal{F}_0}([-2\tau,0],R^n),\quad t\in[-2\tau,0],\end{cases} \tag{2}$$

where $x_k(t)=[x_{k1}(t),x_{k2}(t),\ldots,x_{kn}(t)]^{\mathrm{T}}\in R^n$ is the state vector of kth node. $C(r(t))=diag\{c_1^{r(t)},c_2^{r(t)},c_n^{r(t)}\}\in R^{n\times n}$ is a positive diagonal matrix, $A(r(t))=(a_{kl}^{r(t)})_{n\times n}$ and $B(r(t))=(b_{kl}^{r(t)})_{n\times n}$ are respectively described as weight matrix and the delayed weight matrix, $\Gamma=diag(\gamma_1,\gamma_2,\cdots,\gamma_n)\in R^{n\times n}$, $\Gamma=(\Gamma_{kl})_{n\times n}$ denotes the inner coupling matrix of the networks. $f(x_k(t))=[f_1(x_{k1}(t)),f_2(x_{k2}(t)),\ldots,f_n(x_{kn}(t))]^{\mathrm{T}}$ and $g(x_k(t-\tau(t)))=[g_1(x_{k1}(t-\tau(t))),g_2(x_{k2}(t-\tau(t))),\ldots,g_n(x_{kn}(t-\tau(t)))]^{\mathrm{T}}$ represent nonlinear vector-valued functions; $c>0$ is the coupling strength; $G(r(t))=(G_{kl}(r(t)))_{N\times N}$ denotes the outer coupling configuration matrix of the network at mode $r(t)$, which represents the topology structure of the complex network, and the entries $G_{kl}(r(t))$, $k,l=1,2,\ldots,N$ are defined as follows: if there exists a connection between nodes k and $l(l\neq k)$, then $G_{kl}(r(t))=G_{lk}(r(t))>0$, otherwise, $G_{kl}(r(t))=G_{lk}(r(t))=0$ $(j\neq i)$ and the diagonal elements of matrix G are defined by $G_{kk}(r(t))=-\sum_{l=1,l\neq k}^{N}G_{kl}(r(t))=-\sum_{l=1,l\neq k}^{N}G_{lk}(r(t))$.

$\tau(t)$ is the differentiable time-varying delay satisfying $0\leq\tau(t)\leq\tau$, $\dot{\tau}(t)\leq\mu<1$, τ and μ are known scalars. Then, the initial conditions with network systems (1) are given by $e_k(t)=\xi_k(t)\in\mathcal{L}_{\mathcal{F}_0}([-2\tau,0],R^n)$, $t\in[-2\tau,0]$, $k=1,2,\ldots,N$.

In this paper, the transition rates (TR) of the jumping process are considered to be partly uncertain. Assuming the TR matrix for system (2) with s operation modes as follow:

$$[\pi_{rp}]_{s\times s}=\begin{bmatrix}\pi_{11} & ? & \cdots & ?\\ ? & \pi_{22} & \cdots & \pi_{2s}\\ \vdots & \vdots & \ddots & \vdots\\ \pi_{s1} & ? & \cdots & \pi_{ss}\end{bmatrix},$$

where "?" denotes the unknown transition rate. In order to facilitate the application, for $\forall r\in S$ the set S denotes $S=S_1^r\cup S_2^r$ with

$S_1^r=\{p|\pi_{rp}\,is\ known\ for\ p\in S\}$,
$S_2^r=\{p|\pi_{rp}\,is\ unknown\ for\ p\in S\}$.

Then, the response complex delayed networks with stochastic disturbances can be given by

$$\begin{aligned}\dot{y}_k(t)&=-C(r(t))y_k(t)+A(r(t))f(y_k(t))+B(r(t))g(y_k(t-\tau(t)))\\ &\quad+c\sum_{l=1}^{N}G_{kl}(r(t))\Gamma y_l(t-\tau(t))+u_k(t)\\ &\quad+\sigma_k(t,e_k(t),e_k(t-\tau(t)),r(t))\dot{\omega}(t),\quad k=1,2,\ldots,N,\end{aligned} \tag{3}$$

where $y_k(t)=[y_{k1}(t),y_{k2}(t),\ldots,y_{kn}(t)]^{\mathrm{T}}\in R^n$ is the state vector of the ith node of the response complex delayed networks (2) and (3). $\sigma_k(t)$ is the noisy intensity function, $\omega(t)=$