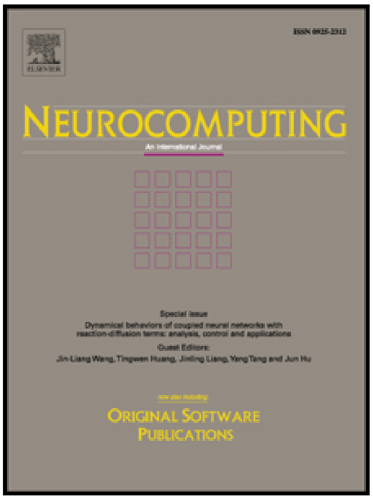
Communicated by Bo Shen

Accepted Manuscript



Multistability of Delayed Neural Networks with Hard-Limiter Saturation Nonlinearities

Mauro Di Marco, Mauro Forti, Massimo Grazzini, Luca Pancioni

PII: S0925-2312(18)30273-X
DOI: 10.1016/j.neucom.2018.03.006
Reference: NEUCOM 19403

To appear in: *Neurocomputing*

Received date: 14 November 2017
Revised date: 5 February 2018
Accepted date: 3 March 2018

Please cite this article as: Mauro Di Marco, Mauro Forti, Massimo Grazzini, Luca Pancioni, Multistability of Delayed Neural Networks with Hard-Limiter Saturation Nonlinearities, *Neurocomputing* (2018), doi: 10.1016/j.neucom.2018.03.006

Multistability of Delayed Neural Networks with Hard-Limiter Saturation Nonlinearities

Mauro Di Marco, Mauro Forti, Massimo Grazzini, Luca Pancioni

Department of Information Engineering and Mathematical Sciences, University of Siena, 53100 Siena, Italy
e-mail: {dimarco, forti, grazzini, pancioni}@dii.unisi.it

Abstract

The paper considers a class of nonsmooth neural networks where hard-limiter saturation nonlinearities are used to constrain solutions of a linear system with concentrated and distributed delays to evolve within a closed hypercube of $\mathbb{R}^n$. Such networks are termed delayed linear systems in saturated mode (D-LSSMs) and they are a generalization to the delayed case of a relevant class of neural networks previously introduced in the literature. The paper gives a rigorous foundation to the D-LSSM model and then it provides a fundamental result on convergence of solutions toward equilibrium points in the case where there are nonsymmetric cooperative (nonnegative) interconnections between neurons. The result ensures convergence for any finite value of the maximum delay and is physically robust with respect to perturbations of the interconnections. More importantly, it encompasses situations where there exist multiple stable equilibria, thus guaranteeing multistability of cooperative D-LSSMs. From an application viewpoint the delays in combination with the property of multistability make D-LSSMs potentially useful in the fields of associative memories, motion detection and processing of temporal patterns.

Keywords: Delayed neural networks; Nonsmooth dynamical systems; Cooperative systems; Multistability; Hard-limiter saturation nonlinearities.

1. Introduction

A neural network (NN) is said to be multistable if there coexist multiple stable equilibrium points (EPs) and the generic solution converges toward an EP depending on the initial condition [1, 2]. The importance of convergence and multistability cannot be overemphasized. Multistable NNs are well suited for instance to implement addressable memories. Here, each stable EP is a pattern memorized by the NN and, given an initial condition containing some partial information on a pattern, the NN is able to retrieve the whole information during the transient motion toward the EP. This association between initial conditions and patterns is crucial also for solving several other classes of pattern recognition, decision and signal processing problems in real time [1, 3, 4, 5, 6].

It is worth mentioning that multistability is in sharp contrast to global asymptotic stability, where the NN has a unique EP where all solutions are attracted independently of the initial conditions, and that globally stable NNs are useful in a different application field as the solution of global optimization problems in real time [7, 8, 9, 10, 11].

Convergence and multistability have been widely investigated over the last decades for diverse NN models as Cohen-Grossberg NNs, Hopfield NNs, cellular NNs, and many others, see [12, 13, 14] and references therein, and these topics are currently receiving an ever increasing interest as witnessed by several publications in recent years (e.g., [15, 16, 17, 18, 19, 20, 21, 22, 23, 24]).

In the fundamental paper [4], Li, Michel and Porod introduced a class of nonsmooth NNs where ideal hard-limiter *saturation nonlinearities* are used to constrain the state evolution of a linear system within the closed hypercube $K = [-1, 1]^n$ of $\mathbb{R}^n$. Such networks, referred to as linear systems in saturated mode (LSSMs), can be considered as modified/improved Hopfield NNs and have been originally proposed for overcoming some problems in the analysis, synthesis and implementation of Hopfield NNs.

First of all it has been shown in [4, Sect. IV] that it is possible to implement an effective algorithm for finding the location and stability properties of all EPs of an LSSM by decomposing K in 3^n subsets where an LSSM reduces to a linear system of equations. By means of Lyapunov method, and the techniques for studying the location and stability of EPs, [4] has established results on convergence and multistability of LSSMs with a symmetric interconnection matrix and devised a systematic synthesis procedure for associative memories [25], which is widely applied, and is also implemented in a MATLAB toolbox [26]. For comparison it is known that it is instead in general difficult to find the number and locations of the EPs, and hence to synthesize equilibria and check the performance, of Hopfield NNs with sigmoid activations. Furthermore, as discussed in [4, 27, 28], the limited range $[-1, 1]$ of the LSSM state variables permits to obtain relevant advantages in the electronic VLSI implementation of LSSMs, namely, lower power consumption, higher cell densities, and increased processing speed, compared to Hopfield NNs.

A model that is closely related to LSSMs, using hard-limiter nonlinearities for constraining the state evolution in a hypercube of $\mathbb{R}^n$, is that of the full-range cellular neural networks (FRCNNs) [29, 6, 30, 31]. FRCNNs have relevant advantages