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Dynamics of four-neuron recurrent inhibitory loop with state-dependent time delays

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ABSTRACT

In previous works, recurrent inhibitory loops with state-dependent propagation delays, firing process and absolute refractory period have been successfully employed to solve capacity-simplicity dilemma in associative memory attractors networks. But in realistic networks, usually there are more than two or three neurons. In order to disclose their natures, in this paper, we consider the dynamics of periodic patterns mainly in a four-neuron recurrent inhibitory loop. We explicitly address how to give rise to its enormous periodic patterns and obtain their existence conditions. At last, we further execute numerical simulations to address the difference from the five-neuron loop and demonstrate similar periodic patterns. Even for smaller τ, the coexistence of three periodic patterns is found in four-neuron loop and that of four periodic patterns is discovered in five-neuron loop. New periodic patterns are also generated and the maximum values of τ for the existence of possible periodic patterns also decrease with the increment of the number of neurons in loops. They will improve loops performance greatly. Our research shows that delays are significant and remarkable for the dynamics of recurrent inhibitory loops, since loops with more neurons have more complicated dynamical behavior and loops performance are also enhanced due to the the increase of synaptic delay and propagation delay.

1. Introduction

There are plenty of neurons in a living nervous system and recurrent inhibitory loops involving two or more neurons are ubiquitous [1]. Except for the study of the neural network or the biological neural network, it also has been simulated to realize some performances of the living nervous system. The nervous network has a wide range of applications including pattern recognition [2,3], signal processing [4], associative memory [5], knowledge engineering, expert systems, optional regrouping [6], robot control and so on [7]. Also it has been employed successfully to develop the neuromorphic computers, i.e., the computers with integrated biological neurons, by researchers [8–11] and by some companies, such as IBM [12]. Especially, deep learning is rooted in and supported to develop greatly by neural networks successfully by Z. Wang et al. [13,14], Fan et al. [15], Schmidhuber [16], Cha et al. [17], Ghazi et al. [18], Sun et al. [19] and Jia et al. [20], Ronao and Cho [21], Kelley et al. [22], Shin et al. [23], Cha et al. [24], Hu et al. [25]. An excellent review of deep neural network architectures and their applications was conducted and applications of deep learning techniques on some selected areas (speech recognition, pattern recognition and computer vision) were highlighted [13]. For the first time, as one of the latest methodologies in machine learning, the deep belief network was applied to quantitative analysis of GICS images [14]. At the same time, the bio-inspired computing models or the living neurons' hardware including the artificial neural networks are the emerging tendency and tool [26–29]. In the future, we will consider these models to improve the performance of the neural networks.

In general, when a neuron excites, nerve impulse is delivered from the axons of the neuron to the dendrites of other neurons. Delays are intrinsic properties of nervous systems and are unavoidable in electronic implementation due to distances among neurons, axonal conduction times and the finite switching speeds of amplifiers [30]. Effects of delays on the dynamics of neural networks have been investigated in many works [31–34]. In fact, dynamics of delayed neural networks is very complex and complicated bifurcation may occur when parameters vary [35,36]. So it is interesting to investigate periodic patterns of delayed neural networks as in [37] and [38]. We all know that the action potential of a neuron will rise when delivering excitatory signal and drop when

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delivering inhibitory signal. However the effective time of up or down is uncertain because of the existence of the firing process period and the absolute refractory period. Therefore, propagation delays corresponding to different periodic patterns are significantly important.

Periodic patterns exhibiting in neural networks have been linked to a variety of rhythms, which are associated with important behavioral and cognitive states in the nervous system, including attention, working memory, associative memory, object recognition, sensory motor integration and perception processing. Thus, to study the neuron dynamics of the realistic models, many works have focused on the multi-stability in delayed neural networks, especially in delayed recurrent inhibitory loops (see [37–39]) where the capacity-simplicity dilemma in associative memory attractor networks have been solved successfully.

In this paper, incorporating the firing process and the absolute refractory period, we investigate the delayed recurrent inhibitory systems mainly with four neurons and locate the coexistence of multiple periodic patterns in them due to different state dependent propagation delays and synaptic delay. Based on the works of the two-neuron and three-neuron recurrent inhibitory systems (see [37–39]), we introduce propagation delays to indicate the inhibitory post-synaptic potential as a feedback of a single neuron and aim to investigate how the number of neurons affects the dynamics of system in the recurrent inhibitory loop since usually there are no less than 2 or 3 neurons in the realistic loops. We also display that the interaction can generate four types of basic oscillations exhibited by neuron E_1, which are the basic building blocks of periodic patterns. More precisely, we explore periodic patterns composed of only W_u and $W_d - oscillations$ in Section 4.1 and self-inhibitory periodic patterns in Section 4.2 and other kinds of periodic patterns in Sections 4.3 and 4.4. Furthermore, we gain the coexistence of multiple stable periodic patterns and their bifurcations with the varying synaptic delay. And we successfully demonstrate our theoretical results by numerical simulations. But due to the increasing computational complexity, for the five-neuron system, only numerical simulations are provided to show the existence and coexistence of some periodic patterns and furthermore to indicate the more complicated dynamical behavior for systems with more neurons (see supplementary materials).

The rest of this paper is organized as follows. In Section 2, we formulate the integrate-and-fire model for the recurrent inhibitory loop with three excitatory neurons and one inhibitory neuron. We also exhibit that the loop can generate four types of basic oscillations. In Section 3, we introduce the propagation delays to indicate the inhibitory post-synaptic potential as a feedback of neuron E_1 and discuss the general principles on how these basic oscillations interact to generate the periodic patterns of E_1. In Section 4, the theoretical results are established for periodic patterns of E_1. Numerical simulations and conclusion are presented in Sections 5 and 6, respectively. Numerical simulations of the five-neuron system are presented in Supplementary Materials. For the clarity of the computation in this paper, the flow chart is presented in Fig. 1.

2. Model and its basic oscillatory patterns

2.1. The model of recurrent inhibitory loop

Consider the four-neuron recurrent inhibitory loop in Fig. 2.

This loop consists of four neurons E_1, E_2, E_3 and I, where E_1, E_2, E_3 are excitatory and I is inhibitory. The basic neural signal processing of such a loop is described by the integrate-and-fire model, and hence the temporal evolution of the action potentials $V_{E_i}, i = 1, 2, 3$ and V_I of neurons is governed by the following system,

$$\begin{cases} \dot{V}_{E_1}(t) = -V_{E_1}(t) - F_I(t) + I_0, \\ \dot{V}_{E_2}(t) = -V_{E_2}(t) + F_{E_1}(t), \\ \dot{V}_{E_3}(t) = -V_{E_3}(t) + F_{E_2}(t), \\ \dot{V}_I(t) = -V_I(t) + F_{E_3}(t), \end{cases} \tag{1}$$

where I_0 is the stimulus (assumed to be a constant), $F_{E_1}(t)$ describes the excitatory feedback from neuron E_1 to E_2, $F_{E_2}(t)$ describes the excitatory feedback from neuron E_2 to E_3, $F_{E_3}(t)$ describes the excitatory feedback from neuron E_3 to I, $F_I(t)$ describes the inhibitory feedback from neuron I to E_1. The first term in the right hands of the equations in system (1) determines the effective timing of the feedback in the absolute refractory period, a short period after the firing of a spike during which the neuron is not affected by inputs at all. The absolute refractoriness allows the membrane potential to decay back from the hyperpolarization (afterpotential) to the resting potential even if feedback is delivered during this period. The second term in the right hands in system (1) describes the excitatory or inhibitory feedback from the last neuron, which makes the action potentials rise or fall.

For E_1, E_2, E_3, I, once their potentials reach the given threshold ϑ, the fires follow a pattern governed by the following piecewise linear function,

$$V_f(t) = \begin{cases} \vartheta + \frac{c-\vartheta}{s_1}(t - t_f) & \text{if} \quad t \in [t_f, t_f + s_1), \\ V_r + \frac{c-V_r}{T_F - s_1}(t_f + T_F - t) & \text{if} \quad t \in [t_f + s_1, t_f + T_F], \end{cases}$$

where c is the peak value and $c > \vartheta$, V_r is the reset potential ($V_r < 0$), T_F is the spike width, t_f is the firing time. Notice that the action potential increases from ϑ to c in the interval $[t_f, t_f + s_1)$ and then decays to V_r in the interval $[t_f + s_1, t_f + T_F]$. t_f satisfies the threshold condition, namely,

$$t_f : V(t_f) = \vartheta \text{ and } V(t_f - \varepsilon) < \vartheta \quad \text{for any small } \varepsilon > 0.$$

There is an absolute refractory after the firing process and the action potential is given by

$$V_f(t) = V_r e^{-(t - t_f - T_F)} + E[1 - e^{-(t - t_f - T_F)}] \quad \text{for } t \in [t_f + T_F, t_f + T_{FR}],$$

where E is a constant and $E > 0$. $T_{Re} := T_{FR} - T_F$ is the duration of the absolute refractory period, so the sum of the firing time and the absolute refractory time is T_{FR}. V_A is the after-potential at time $t_f + T_{FR}$, namely,

$$V_A = V_r e^{-T_{Re}} + E(1 - e^{-T_{Re}}).$$

We assume $V_A < 0$, which implies $V_r < V_A < 0$. The action potential increases from V_r to V_R in the interval $[t_f + T_F, t_f + T_{FR}]$. Any external input or internal feedback has no effect on the action potentials during the firing time and the absolute refractory time.

The excitatory feedback functions are represented respectively by the following,

$$F_{E_1}(t) = \begin{cases} b_1 & \text{if } t \in [t^\tau_{f,E_1} + \tau, t^\tau_{f,E_1} + \tau + T_{EF}], \\ 0 & \text{otherwise}, \end{cases}$$

$$F_{E_2}(t) = \begin{cases} b_2 & \text{if } t \in [t^\tau_{f,E_2} + \tau, t^\tau_{f,E_2} + \tau + T_{EF}], \\ 0 & \text{otherwise}, \end{cases}$$

$$F_{E_3}(t) = \begin{cases} b_3 & \text{if } t \in [t^\tau_{f,E_3} + \tau, t^\tau_{f,E_3} + \tau + T_{EF}], \\ 0 & \text{otherwise}, \end{cases}$$

where T_{EF} is the duration of the excitatory feedback (EFB) and t^τ_{f,E_i} is the last firing time of excitatory neuron E_i prior to the time $t - \tau, i = 1, 2, 3$, τ is the synaptic delay.