Neurocomputing 000 (2018) 1–7

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Global synchronization of fractional complex networks with non-delayed and delayed couplings

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A R T I C L E I N F O

Article history:
Received 6 October 2017
Revised 5 January 2018
Accepted 5 February 2018
Available online xxx

Communicated by Dr. Jin-Liang Wang

Keywords:
Fractional complex network
Global synchronization
Delay
Razumikhin theorem

A B S T R A C T

This paper deals with global synchronization of fractional complex networks with non-delayed and delayed couplings. Applying fractional Razumikhin theorem, a simple quadratic Lyapunov function is constructed and two linear matrix inequality (LMI) criteria on global synchronization are proposed. It is very convenient and efficient to check synchronization of the presented network models by using our proposed method. Finally, numerical simulations are given to show the efficiency of the obtained results.

1. Introduction

During recent years, complex networks have attracted more and more attention due to the important and wide applications in various fields, many practical complex systems can be modeled by complex dynamical networks, such as World Wide Web [1], biological networks [2], gene networks [3], neural networks [4–6] and so on. Synchronization as a most significant dynamical behavior among complex networks, has been a hot research topic and attracted increasing interest, a large number of important and meaningful works have been obtained [7–12]. However, the above works are concerned with integer-order dynamical networks, many researches show that fractional derivative provides a more excellent instrument for the description of various materials and processes in comparison with classical integer-order derivative [13–15]. Recently, various kinds of synchronization including adaptive synchronization [16,17], pinning synchronization [18], global synchronization [19], projective synchronization [20,21] and impulsive synchronization [22,23] have been extensively investigated.

On the other hand, it should be mentioned that time delay is a universal phenomenon in many practical dynamical systems, time delay has important effect on dynamical behavior of systems and even make system unstable. Due to the finite information transmission and processing speeds among nodes, time delay coupling will unavoidably occur in complex networks. It is well known that Lyapunov stability theory is a most efficient approach to analyze stability and synchronization of complex dynamical systems. In [24], Liu et al. present an important inequality on Riemann–Liouville derivatives of quadratic functions which provides a very useful and easy way to analyze stability of fractional systems. Based on the method presented in [24], several stability criteria on fractional neutral and singular systems with delays are obtained [25,26], Zhang et al. [27] investigate stability of a class of neutral delayed neural networks, several sufficient conditions are obtained in terms of matrix inequalities. However, in Caputo sense, stability analysis of fractional systems with time delay makes little progress since the composition property of Caputo calculus does not hold, that is, ${}_{t_0}D_t^p({}_{t_0}D_t^q f(t)) \neq {}_{t_0}D_t^{p+q} f(t)$, where ${}_{t_0}D_t^{\alpha}$ denotes Caputo derivative if $\alpha > 0$ and denotes Riemann–Liouville integral if $\alpha < 0$, which brings great difficulty in studying synchronization of Caputo fractional complex networks with delayed coupling. At present, comparison principle is an important instrument to handle synchronization problems of fractional complex networks with delayed coupling, for example, Liang et al. [28] investigate adaptive pinning synchronization in fractional-order uncertain complex dynamical networks with delay, Wang et al. [29] discuss global stability of fractional-order Hopfield neural networks with time delay, projective synchronization of fractional-order memristive neural networks with switching jumps mismatch is studied in [30]. It is well known that that the key issue of using Lyapunov direct method is to select a suitable Lyapunov function for the considered system. As far as we know, in Caputo sense, how to choose a proper Lyapunov function and obtain useful algebraic criteria on

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https://doi.org/10.1016/j.neucom.2018.02.026
0925-2312/

stability and synchronization for a fractional nonlinear delayed system is still an open and extremely challenging problem.

Motivated by the above results, this paper aims to investigate Caputo fractional complex networks with non-delayed and delayed couplings. Applying fractional Razumikhin theorem, a simple quadratic Lyapunov function is constructed and two linear matrix inequality (LMI) criteria on global synchronization are proposed. It is very convenient and efficient to check synchronization of the presented network models by using our proposed method. Moreover, the approach presented in this paper can be applied to general fractional complex networks with non-delayed coupling or with delayed coupling. Finally, numerical simulations are given to show the efficiency of the obtained results.

2. Preliminaries

In this section, some basic definitions of fractional calculus and important lemmas are introduced which will be needed later.

Definition 2.1 [31]**.** The p-order Reimann–Liouville fractional integral is defined as

$$ {}_{t_0}D_t^{-p}x(t) = \frac{1}{\Gamma(p)}\int_{t_0}^{t}(t-s)^{p-1}x(s)ds, \quad p>0. \tag{2.1}$$

Definition 2.2 [31]**.** The q-order Caputo fractional derivative is defined as

$$ {}_{t_0}^{C}D_t^{q}f(t) = \frac{1}{\Gamma(n-q)}\int_{t_0}^{t}\frac{f^{(n)}(s)}{(t-s)^{q-n+1}}ds, \quad n-1<q\le n. \tag{2.2}$$

Lemma 2.1 [32]**.** *Let $x(t)\in\mathbf{R}^n$ be a vector of differentiable function. Then the following relationship holds*

$$\frac{1}{2}\,{}_{t_0}^{C}D_t^{q}(x^T(t)Px(t)) \le x^T(t)P\,{}_{t_0}^{C}D_t^{q}x(t), \quad 0<q<1, \tag{2.3}$$

where $P\in\mathbf{R}^{n\times n}$ is a constant, square, symmetric and positive semi-definite matrix.

Lemma 2.2 [33]**.** *For any $x,\ y\in\mathbf{R}^n, \varepsilon>0$, the inequality $2x^Ty\le \varepsilon x^Tx+\frac{1}{\varepsilon}y^Ty$ holds.*

Let $C =: C([-r,0],\mathbf{R}^n)$ denote the space of continuous functions on $[-r,0]$.

Consider the following fractional delayed system

$$ {}_{t_0}^{C}D_t^{q}x(t) = f(t,x_t), \quad t\ge t_0, \tag{2.4}$$

where $0<q\le 1$ and $x_t(\theta)=x(t+\theta),\ \theta\in[-r,0]$.

Lemma 2.3 [34]**.** *Suppose that f in (2.4) maps $\mathbf{R}\times$(bounded sets of $\mathcal{C}$) into bounded sets in $\mathbf{R}^n$, and u, v, w: $\mathbf{R}^+\to\mathbf{R}^+$ are continuous nondecreasing functions, $u(s)$ and $v(s)$ are positive for $s>0$, and $u(0)=v(0)=0$, v is strictly increasing. If there exist a continuous function V: $\mathbf{R}\times\mathbf{R}^n\to\mathbf{R}$ and a continuous nondecreasing function $p(s)>s$ for $s>0$ such that for $t\in\mathbf{R}$, $x\in\mathbf{R}^n$ and $\varphi\in C$,*

$$u(\|x\|)\le V(t,x)\le v(\|x\|), \tag{2.5}$$

and

$$\begin{aligned} {}_{t_0}^{C}D_t^{q}V(t,\varphi(0))\,|_{(2.4)} &\le -w(\|\varphi(0)\|) \text{ if } V(t+\theta,\varphi(\theta)) \\ &\le p(V(t,\varphi(0))), \end{aligned} \tag{2.6}$$

for $\theta\in[-r,0]$ and $t\ge t_0$, then the equilibrium point $x=0$ of the system (2.4) is asymptotically stable. If $u(s)\longrightarrow\infty$ as $s\longrightarrow\infty$, then the solution $x=0$ is also a global attractor.

3. Main results

In this section, we will present two sufficient conditions on global synchronization of fractional complex network with non-delayed and delayed couplings, the criteria are described in terms of linear matrix inequalities.

Consider the following fractional complex network

$$\begin{aligned} {}_{t_0}^{C}D_t^{q}x_i(t) &= f(x_i(t),t)+\sum_{j=1}^{N}c_{ij}Ax_j(t) \\ &\quad +\sum_{j=1}^{N}\hat{c}_{ij}\hat{A}x_j(t-\tau), \quad i=1,2,\ldots,N, \end{aligned} \tag{3.1}$$

where $0<q<1$, $x_i(t)=(x_{i1}(t),x_{i2}(t),\ldots,x_{in}(t))^T\in\mathbf{R}^n$ is the state vector of the ith node, $\tau>0$ is a constant delay. A and $\hat{A}\in\mathbf{R}^{n\times n}$ are the inner coupling matrices, and $C=(c_{ij})_{N\times N}$ (the entries of $\hat{C}=(\hat{c}_{ij})_{N\times N}$ are defined similarly) are the coupling configuration matrices, in which c_{ij} is defined as follows: if there is a direct connection from node i to j, then $c_{ij}>0$; otherwise, $c_{ij}=0$, and the diagonal elements of matrix C are defined by $c_{ii}=-\sum_{j=1,j\neq i}^{N}c_{ij}=-\sum_{j=1,j\neq i}^{N}c_{ji}$.

Suppose that network (3.1) is connected, hence, matrices C and $\hat{C}$ are irreducible, which imply zero is the single eigenvalue of C and $\hat{C}$, and the corresponding eigenvector is a scalar multiple of all one vector. Moreover, the eigenvalues of C and $\widehat{C}$ can be arranged as, respectively

$$0=\lambda_1>\lambda_2\ge\lambda_3\ge\cdots\ge\lambda_N, \qquad 0=\mu_1>\mu_2\ge\mu_3\ge\cdots\ge\mu_N. \tag{3.2}$$

Definition 3.1. The fractional complex network (3.1) is said to achieve globally asymptotical synchronization if

$$\lim_{t\to\infty}\|x_i(t)-x_j(t)\|=0, \quad \text{for any } i,j=1,2,\ldots,N, \tag{3.3}$$

for any initial conditions.

Let $\bar{x}(t)=\frac{1}{N}\sum_{j=1}^{N}x_j(t)$, $e_i(t)=x_i(t)-\bar{x}(t), i=1,2,\ldots,N$, one has from (3.1)

$$\begin{aligned} {}_{t_0}^{C}D_t^{q}e_i(t) &= f(x_i(t),t)-\bar{f}+\sum_{j=1}^{N}c_{ij}Ae_j(t) \\ &\quad +\sum_{j=1}^{N}\hat{c}_{ij}\hat{A}e_j(t-\tau), \quad i=1,2,\ldots,N, \end{aligned} \tag{3.4}$$

where $\bar{f}=\frac{1}{N}\sum\limits_{j=1}^{N}f(x_j(t),t)$.

To make system (3.1) achieve global synchronization, we need the following assumption.

Assumption 3.1. Assume that there exist nonnegative constants L_{ij} such that

$$\begin{aligned} &\|f(x_i(t),t)-f(x_j(t),t)\| \\ &\le L_{ij}\|x_i(t)-x_j(t)\|, \quad i\neq j,\ i,j=1,2,\ldots,N. \end{aligned} \tag{3.5}$$

Theorem 3.1. *Suppose that $C\hat{C}$ is symmetric and Assumptions 3.1 holds, then the network (3.1) will achieve global synchronization if there exist positive definite, symmetric matrices P_i and some scalar $\sigma>1$ such that*

$$\mu_i^2P_i\hat{A}P_i^{-1}\hat{A}^TP_i+\lambda_iP_iA+\lambda_iA^TP_i+\sigma P_i+\eta_iI<0, \quad i=2,3,\ldots,N, \tag{3.6}$$

where $\eta_i>\frac{4L}{N}(N-1)\|P_i\|$, $L=\sum_{i=1}^{N}L_i$, $L_i=\sum_{j=1,i\neq j}^{N}L_{ij}$.

Proof. First, (3.4) can be rewritten in the following compact form £º

$$ {}_{t_0}^{C}D_t^{q}e(t)=F(e(t),t)+Ae(t)C^T+\hat{A}e(t-\tau)\hat{C}^T, \tag{3.7}$$

where $e(t)=(e_1(t),e_2(t),\ldots,e_N(t))$, $F(e(t),t)=(f(x_1(t))-\bar{f},f(x_2(t))-\bar{f},\ldots,f(x_N(t))-\bar{f})$.