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Distributed leader-following consensus of nonlinear multi-agent systems with nonlinear input dynamics

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ABSTRACT

This paper addresses the problem of distributed leader-following consensus for a multi-agent system with an affine nonlinear term. The communication topology we adopt is an undirected connected graph and the leader sends its information to one or more followers. To make each follower asymptotically synchronize with the leader, a nonlinear distributed control protocol is proposed. Using a Lyapunov function and a matrix theory, we establish sufficient conditions which ensure the consensus of these nonlinear multi-agent systems. Finally, a numerical simulation is provided to verify the effectiveness and usefulness of the developed method.

1. Introduction

Inspired by natural biological consensus behaviors, considerable attention has been extensively paid to the consensus of agents by virtue of its widespread range of applications in many areas, such as robots, sensor networks, biological systems, formation and cooperative control, just to name a few [1–6]. As the most essential and important problems, consensus problems for multi-agent systems have drawn attention greatly in recent years [7–10]. From first-order systems [7], linear systems [8] to high-order systems [9], and then, nonlinear systems [10], the researches on consensus control are getting deeper. The consensus is considered to be achieved if each agent in the network converges to a certain common value. A principal problem for the consensus control is to construct effective control algorithms and laws based on the neighbor information to make some agents reach consensus under the corresponding topology.

In general, the existing results about consensus control concentrate on two types of control problems, namely leaderless consensus and leader-following consensus. The former requests that each agent converges to a certain agreement state [11–13], which is related to the initial conditions. For example, the consensus problems for a network of first-order agents are addressed in [11]. Each agent eventually converges to the average of the initial states under balanced digraphs. Recently, consensus of multi-agent systems with a leader has been extensively studied [14–17], whose goal is to guarantee that a group of agents can track the state trajectory of a leader. In [14], the authors concern the consensus for first-order networks with a time-varying leader. The study in [15] is an extension of [14] from first-order to high-order systems. In [16], the semi-global leader-following consensus is investigated for linear systems, whose actuators are imperfect. In [17], global leader-following consensus is discussed under bounded controls. It is true that with the help of a leader, the application range is enlarged for multi-agent systems.

Till now, many results on simple linear multi-agent systems can be found in the open literature, including most references mentioned above. However, in practice, most of engineering problems involve complex nonlinear systems so that results for linear systems do not apply. Hence, the nonlinearities in dynamics have been taken into consideration for many researchers recently. In [18], the authors concern the consensus of third-order multi-agent systems with nonlinear dynamics under a fixed topology. In [19], tracking consensus is studied for nonlinear multi-agent systems. As an extension, the switching networks and unreliable communication are further discussed in [20].

Compared with results in [19,20], the affine nonlinear term is further introduced for the nonlinear systems in this paper. There is no doubt that affine nonlinear systems can simulate many actual systems in many fields, including mechanics, physics, ecology, and engineering [21–23]. The stability analysis of affine nonlinear

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systems is given in [24]. To the best of our knowledge, very few results on consensus problem of affine nonlinear systems can be found in the open literature. For example, the consensus problem is addressed for nonlinear multi-agent systems in [25], which are affine in control. However, there is no research on the tracking consensus for affine nonlinear systems. Based on the discussion above, a type of nonlinear systems with the affine nonlinear term are proposed and the leader-following consensus is introduced to them in this work. Motivated by our previous works on nonlinear controllers design for nonlinear systems [26,27], we develop a numerical method for leader-following problem involving a type of affine nonlinear systems. More specifically, we construct a novel nonlinear control algorithm for each follower agent, under which sufficient conditions are obtained for reaching consensus. We assume that the followers communicate with the undirected and connected graph, and the leader sends its message to one or more followers. The key contributions of the paper are threefold. (1) Compared with the existing conclusions, this work can handle a class of more general nonlinear systems with affine nonlinear term. (2) The tracking consensus is investigated for the nonlinear systems. (3) The relationship between design parameters and consensus performance has been built and revealed to guarantee the consensus.

The overall structure of the rest of the paper is as follows. A problem statement and preliminaries, which include some definitions, assumptions and some other basic graph knowledge, are illustrated in Section 2. In Section 3, we design a nonlinear control protocol for each follower, and sufficient conditions are provided to ensure the consensus of the system proposed. In Section 4, we present some simulation results to demonstrate the effectiveness and usefulness of our method by using a non-trivial test example. Section 5 makes a brief summary finally.

Notation. The following notations are applied throughout the paper. For given positive integers m and n, R^n stands for the Euclidean space of n dimensional, and $R^{m\times n}$ represents a set of $m\times n$ matrices. We use 1_N to denote the a column vector in R^N whose entries are all equal to one. I_N means the $N\times N$ identity matrix. $\|\cdot\|$ is the Euclidean norm of a certain vector. If not specifically pointed out, the dimensions of each matrix is supposed to be appropriate. Furthermore, we use $diag\{\cdot\}$ to denote a matrix, whose diagonal are the corresponding parameter values, and other entries are all zero. The transpose of a matrix A is symbolled by A^T and $\otimes$ is the Kronecker product, which has many properties, such as $(A\otimes B)(C\otimes D)=AC\otimes BD$ and $(A\otimes B)^T=A^T\otimes B^T$. For a square matrix P, $P>0$ or $P<0$ represents it is a positive or negative definite matrix respectively.

2. Problem statement and preliminaries

In this section, we give a brief account of some graph theory basis and present the leader-following consensus problem.

2.1. Graph theory

Graphs are often applied to model the communication rules among the agents. Take a graph $G=\{\nu,\varepsilon,A\}$ with N nodes as example, N nodes (follower agents) are represented by $\nu=\{\nu_1,\nu_2,\ldots,\nu_N\}$, and the leader is labeled as ν_0. All edges or arcs are denoted as $\varepsilon\subseteq\nu\times\nu$. The pair $(\nu_j,\ \nu_i)\in\varepsilon$ means there is an edge connecting the agents j and i. Agent j is known as a neighbor of agent i if $(\nu_j,\ \nu_i)\in\varepsilon$. The structure of G can also be described by the adjacency matrix $A=\left[a_{ij}\right]$. If agent i can get information from agent j, $a_{ij}>0$, otherwise $a_{ij}=0$. We suppose that $a_{ii}=0$ for all $i=1,2,\ldots,N$. The in-degree of agent i is denoted as $d_i=\sum_{j=1}^N a_{ij}$, and the in-degree matrix D is a diagonal matrix with $D=diag\{d_i\}\in R^{N\times N}$. Let Laplacian matrix be $L=D-A$. If the leader can send its information to agent i, $a_{i0}>0$ for $i=1,2,\ldots,N$, otherwise $a_{i0}=0$. The graph is said to be undirected if $a_{ij}=a_{ji}$, i.e., the agent i and j can communicate with each other. If there is an undirected path between any two vertices, the undirected graph is called connected. We put $M=L+diag\{a_{10},a_{20},\ldots,a_{N0}\}$.

2.2. Problem statement

Consider N communicated agents with the dynamics of them described as,

$$\dot{x}_i=Ax_i+Bf(x_i)+g(x_i)u_i, \tag{1}$$

where $x_i\in R^n$ is the state vector of agent i, $i=1,2,\ldots,N$, and $u_i\in R^m$ is the control input. $f(x_i(t))\in R^r$ and $g(x_i)=[g_1(x_i),g_2(x_i),\ldots,g_m(x_i)]\in R^{n\times m}$ mean the intrinsic nonlinear dynamics. $A\in R^{n\times n}$ and $B\in R^{n\times r}$ are both constant system matrices.

The dynamics of the leader is described by the following nonlinear form,

$$\dot{x}_0=Ax_0+Bf(x_0), \tag{2}$$

where $x_0\in R^n$ is the state of the leader. Its state trajectory evolves without being effected by the followers, and the leader provides the information for the followers to track its trajectory.

Definition 1. [28] For any initial condition $x_i(0)$, $i=0,1,2,\ldots,N$, the leader-following consensus of the system (1)–(2) is achieved if there exists a control input u_i that makes the system satisfy the following relationship:

$$\lim_{t\to\infty}\|x_i(t)-x_0(t)\|=0,\ i=1,2,\ldots,N.$$

The communication topology connecting the agents is considered to satisfy the assumption as below.

Assumption 1. The undirected topology G we adopt is connected, and the leader sends its information to one or more followers.

Remark 1. Some literature have investigated the consensus with directed connected graphs, such as [29,30] for continuous-time systems, and [31] for discrete-time systems. In the future work, more general communication networks will be taken into consideration.

Assumption 2. For a constant $\rho>0$, the nonlinear dynamics $f(x)$ satisfies the Lipschitz condition as follows,

$$\|f(a)-f(b)\|\le\rho\|a-b\|,\ \forall a,b\in R^n.$$

Assumption 3. For constant system matrix A, there exists a positive matrix $P>0$, which satisfies the equation $A^TP+PA=Q$ for some $Q<0$.

Remark 2. If Assumption 3 is satisfied, an appropriate positive matrix P can be obtained by condition (4).

3. Main results

In this section, we design the following distributed control protocol for each follower,

$$u_i(t)=-g^T(x_i(t))P\left(\sum_{j=1}^N a_{ij}\left(x_i-x_j\right)+a_{i0}(x_i-x_0)\right), \tag{3}$$

in which a_{ij}, the elements of matrix A, means the communication weight among the followers, a_{i0} is the weight between the leader and each follower agent, and P is a positive matrix.

Theorem 1. *Consider the followers and the leader whose dynamics are respectively described by (1) and (2). Suppose that Assumptions 1,*