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A new Mumford–Shah total variation minimization based model for sparse-view x-ray computed tomography image reconstruction

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ABSTRACT

Total variation (TV) minimization for the sparse-view x-ray computer tomography (CT) reconstruction has been widely explored to reduce radiation dose. However, owing to the piecewise constant assumption, CT images reconstructed by TV minimization-based algorithms often suffer from image edge oversmoothness. To address this issue, an improved sparse-view CT reconstruction algorithm is proposed in this work by incorporating a Mumford–Shah total variation (MSTV) model into the penalized weighted least-squares (PWLS) scheme, termed as "PWLS-MSTV". The MSTV model is derived by coupling TV minimization and Mumford–Shah segmentation, to achieve good edge-preserving performance during image denoising. To evaluate the performance of the present PWLS-MSTV algorithm, both qualitative and quantitative studies were conducted by using a digital XCAT phantom and a physical phantom. Experimental results show that the present PWLS-MSTV algorithm has noticeable gains over the existing algorithms in terms of noise reduction, contrast-to-ratio measure and edge-preservation.

1. Introduction

X-ray computed tomography (CT) has been widely used in clinical applications over the past decades. However, excessive x-ray radiation exposure during clinical examination has been concerned about increasing lifetime risk of cancerous, genetic, and other diseases [1–4]. Therefore, minimizing the radiation risks is strongly desirable in clinical practices. To reduce the radiation dose, two major strategies have been widely discussed, including reducing the milliampere-seconds per projection view or decreasing the required number of projection views (sparse-view) per rotation around the body [5–7]. In this work, we are focusing on low-dose CT image reconstruction from the spare-view projection data. In the sparse-view CT scans, less projection views will unavoidably lead the acquired sinograms insufficient and increase the data inconsistence associated with the sparsity. And this data inconsistence would cause image artifacts. Therefore, the diagnostic quality of the CT images would be degraded if appropriate methods are not applied during image reconstruction.

To address the issues of sparse-view CT image reconstruction, various image reconstruction methods by incorporating adequate prior information of the desired image have been proposed [8–14]. A typical example is total variation (TV) based projection onto convex sets (POCS) reconstruction strategy, based on the piecewise constant assumption of the desired image, has shown its effectiveness for dealing with the data insufficiency from sparse-view sampling [8–10]. Furthermore, to address the limitations of the original TV constrain with isotropic edge property, different weighted-TVs were proposed recently [11–16].

In this work, aiming to improve the performance of TV minimization based algorithm, we introduce a Mumford–Shah TV (MSTV) minimization for sparse-view CT image reconstruction under the penalized weighted least-squares (PWLS) criteria, which is termed as "PWLS-MSTV". The MSTV regularization presents an integrated framework for TV minimization and image segmentation, which reduces the noise and artifacts in the segments without over-smoothing the edges. In the implementation, the PWLS-MSTV is performed by integrating CT image reconstruction and segmentation, aiming to yield a continuous edge map and less noise-induced artifacts. To evaluate the present PWLS-MSTV

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algorithm, qualitative and quantitative studies were performed on both digital and physical phantoms in terms of noise reduction, image-similarity metric, and convergence analysis.

The remaining part of this paper is organized as follows. Section 2 first describes the MSTV model in brief, and then describes the proposed PWLS-MSTV image reconstruction algorithm in detail and the associative optimization algorithm. In Section 3, experimental setting for both digital and physical phantoms is described. Section 4 is the results and discussion. Finally, the conclusion is given in Section 5.

2. Methods

2.1. Mumford–Shah total variation

The MSTV was first proposed by Shah and used in the image segmentation and image restoration [17–22]. The widely used MSTV is an approximation proposed by Alicandro [22]:

$$MSTV_\varepsilon(u,v)=\int_\Omega v^2|\nabla u|dx+\alpha\int_\Omega\left(\varepsilon|v|^2+\frac{(v-1)^2}{4\varepsilon}\right)dx \tag{1}$$

where Ω is a bounded domain, ∇u is the gradient of image u, v is the edge function of image u, which is approximate to zero in the edge of image u while it is approximate to one in other region of image u, ε is a small positive constant and α is a positive weight which needs to be tuned manually. Alicandro et al. [22] also proved the Γ-convergence of this functional to

$$MSTV(u)=\int_{\Omega\backslash K}|\nabla u|dx+\alpha\int_K\frac{|u^+-u^-|}{1+|u^+-u^-|}dH^1+|D^cu|(\Omega) \tag{2}$$

where u^+ and u^- denote the image values on two sides of the edge set K, H^1 is the one-dimensional Hausdorff measure and D^cu is the Cantor part of the measure-valued derivative Du. Through the definition of MSTV as shown in (2), it is obvious that MSTV not only considered the TV norm of image u in the image domain except for the edge, but also considered the measure of edge set K. Therefore, MSTV regularization brings more powerful regularity of solution than TV regularization.

2.2. PWLS-MSTV for CT image reconstruction

In this study, we propose the following cost function for CT image reconstruction with *MSTV* regularization:

$$\min_{u\geq 0,v}(y-Hu)^TG^{-1}(y-Hu)+\beta_2 MSTV_\varepsilon(u,v) \tag{3}$$

where $G=\frac{1}{\beta_1}HH^T+\Sigma$, β_1 and β_2 are two hyper-parameters to balance these two terms, namely, the fidelity term and the regularization term. u is the vector of attenuation coefficients to be reconstructed, symbol T denotes the matrix transpose. The operator H represents the system or projection matrix with the size of $M\times N$. The element of h_{ij} is the length of the intersection of projection ray i with pixel j. Σ is a diagonal matrix with the ith element of σ_i^2 which is the variance of sinogram data y_i. In this work, the variance σ_i^2 is determined by the following mean–variance relationship based on our previous works [23–25].

Additionally, by introducing a new vector f, we have

$$\min_f(y-Hf)^T\Sigma^{-1}(y-Hf)+\beta_1\|f-u\|^2$$
$$=(y-Hu)^TG^{-1}(y-Hu) \tag{4}$$

Hence, solving formula (4) is equal to solve the below formula:

$$\min_{u\geq 0,f,v}(y-Hf)^T\Sigma^{-1}(y-Hf)+\beta_1\|f-u\|^2+\beta_2 MSTV_\varepsilon(u,v) \tag{5}$$

Namely,

$$\min_{u\geq 0,f,v}(y-Hf)^T\Sigma^{-1}(y-Hf)+\beta_1\|f-u\|^2$$
$$+\beta_2\int_\Omega v^2|\nabla u|dx+\beta_2\alpha\int_\Omega\frac{(v-1)^2}{4\varepsilon}+\varepsilon|\nabla v|^2dx \tag{6}$$

For simplifying the redundant parameter, we replace $\beta_2\alpha$ by l_1 without losing the previous meaning. The above formula is equal to:

$$\min_{u\geq 0,f,v}(y-Hf)^T\Sigma^{-1}(y-Hf)+\beta_1\|f-u\|^2$$
$$+\beta_2\int_\Omega v^2|\nabla u|dx+\alpha\int_\Omega\frac{(v-1)^2}{4\varepsilon}+\varepsilon|\nabla v|^2dx. \tag{7}$$

2.3. Optimization scheme

In order to solve the cost function in (7), a modified alternating optimization method with three minimizing steps, which can be formulated as follows:

$$\begin{aligned}&(P1):f=\arg\min_f(y-Hf)^T\Sigma^{-1}(y-Hf)+\beta_1\|f-u\|^2\\&(P2):u=\arg\min_{u\geq 0}\beta_1\|f-u\|^2+\beta_2\textstyle\int_\Omega v^2|\nabla u|dx\\&(P3):v=\arg\min_v\beta_2\textstyle\int_\Omega v^2|\nabla u|dx+\alpha\int_\Omega\frac{(v-1)^2}{4\varepsilon}+\varepsilon|\nabla v|^2dx\end{aligned} \tag{8}$$

In the implementation, we utilized a separable paraboloidal surrogates (SPS) algorithm [26] to solve (P1), let

$$\Phi(u;f)\triangleq(y-Hf)^T\sum^{-1}(y-Hf)+\beta_1\|f-u\|^2 \tag{9}$$

Since the surrogate is a separable paraboloid, it can be easily minimized by zeroing the first derivative. This leads to the following simultaneous update algorithm:

$$f_j^{k+1}=f_j^k-\frac{\sum_{i=1}^M((1/\sigma_i^2)h_{ij}([Hf^k]_i-y_i))}{\sum_{i=1}^M((1/\sigma_i^2)h_{ij}\sum_{t=1}^N h_{it})+\beta_1}-\frac{\sum_{i=1}^M\beta_1(f_j^k-u_j^k)}{\sum_{i=1}^M((1/\sigma_i^2)h_{ij}\sum_{t=1}^N h_{it})+\beta_1} \tag{10}$$

where the superscript $k=1,2,\ldots,K$ denotes the iteration index. The first and second derivatives of the surrogate are easily shown to be

$$\left.\frac{\partial\Phi(u;f)}{\partial u_j}\right|_{f-u}=\sum_{i=1}^M\left((1/\sigma_i^2)h_{ij}\left(\left[Hf^k\right]_i-y_i\right)\right)+\beta_1\left(f_j^k-u_j^k\right) \tag{11}$$

$$\left.\frac{\partial\Phi(u;f)}{\partial u_j^2}\right|_{f=u}=\sum_{i=1}^M\left(\left(1/\sigma_i^2\right)h_{ij}\sum_{t=1}^N h_{it}\right)+\beta_1 \tag{12}$$

Then focusing on (P2), the solution of (P2) can be obtained by calculate the derivate of the cost functional in (P2), which can be written as follow:

$$2\beta_1(u-f)-2\beta_2 Div\left(v^2\nabla u\right)=0 \tag{13}$$

where Div is the symbol of divergence and assuming $L(v)$ denote the differential operator

$$L(v)=-\beta_2 Div(v^2\nabla u). \tag{14}$$

Then the above Eq. (14) equals to

$$\beta_1(u-f)+\beta_2L(v)=0 \tag{15}$$

Let $A(v)u=\beta_1u+\beta_2L(v)$, by rearranging (15), we can obtain

$$A(v)u=\beta_1 f. \tag{16}$$

As the literature [21], it is shown that the operator $A(v)$ is self-adjoint and positive definite. Therefore, the conjugate gradient