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Synchronization of stochastic coupled systems with time-varying coupling structure on networks via discrete-time state feedback control

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ABSTRACT

This paper is concerned with the exponential synchronization of stochastic coupled systems on networks. Time-varying coupling structure of coupled systems and discrete-time state feedback control are focuses and difficulties of our research. Based on the Lyapunov method and Kirchhoff's Matrix Tree Theorem, two sufficient criteria are obtained, which guarantee the exponential synchronization of stochastic coupled systems with time-varying coupling structure on networks via discrete-time state feedback control. Finally, we apply theoretical results to stochastic coupled oscillators and propose two numerical examples to illustrate the effectiveness and feasibility of the obtained theoretical results.

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1. Introduction

The word "synchronization" comes from Greek, which means "share time". Today, from the point of view of science and technology, it comes to be regarded as a common phenomenon or an adjustment of rhythms of oscillating objects due to their internal weak couplings. It is worth mentioning that the synchronization phenomenon (the synchronous motion of a pendulum hanging on the common base) was first discovered by the Dutch scholar Huygens in the 17th century. And the general definition of synchronization was presented by Blekhman et al. in 1997, see [1]. Nowadays, synchronization has become a hot topic and received a great deal of attention among scientists from various fields, which has been reported in [2–8].

Over the past decades, coupled systems have become very hot, see [9,10]. And the synchronization plays an important role in researching coupled systems as one of their most important dynamic properties, see [11–14]. We notice that coupled systems and their coupling structure were considered in a determinate case in most existing literature. However, coupled systems will inevitably receive the effects of random disturbances from the external environment, see [15–20]. And the coupling structure of coupled systems may be not constant. Some phenomena can be described legitimately if we take the time-varying coupling structure and stochastic disturbances into account. For instance, in biomathematics, the dispersal rate of some species living in different groups varies with time, especially when some natural disasters happen. In epidemiology, the rate of transmission of infectious diseases also changes over time due to the population mobility. Thus, how to guarantee the synchronization of stochastic coupled systems with time-varying coupling structure on networks (SCSTN) is an interesting problem.

In order to synchronize systems, external force controllers usually need to be designed on them. After researchers' long-term exploration, many control strategies have been put forward. For instance, sliding mode control [21–24], H_∞ control [25–28], robust control [29–31] and so on. In [32], Mao studied the mean-square exponential stabilization of the hybrid stochastic differential equations by presenting a new feedback control strategy based on discrete-time state observations. In fact, discrete-time state observations have been applied in various fields because they are convenient and economical. For example, in order to research nonparametric inference problems in the multiplicative intensity model, Negri and Nishiyama proposed a Nelson-Aalen type estimator based on discrete-time observations in [33]. And the results were the same as that of continuous-time observations. Besides, in sampled data control systems, the choice of proper sampling interval is even more important than the design of controllers, see [34,35]. In [36], Barrau and Bonnabel dealt with the

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problem of intrinsic filtering on SO(3) with discrete-time observations. The choice of discrete-time measurements helped them circumvent some technical challenges. In addition, discrete-time observations also have been implemented in geology, engineering and so on. For instance, the analysis and prediction of urban ground settlement are usually based on observed multi-epoch discrete data. Weather stations usually collect discrete-time data to predict future weather, see [37]. It narrows the difficulty of the work and obtains consistent results with continuous-time state observations. So when we study the dynamic properties of systems via feedback control, discrete-time state observations are usually used because they receive the control gain information only at discrete times, and the control cost can be dramatically reduced.

In addition to the external controller, Lyapunov method is also a powerful tool for studying synchronization. In [38], Zhang et al. studied the robust synchronization by using Lyapunov functions and analysis techniques. Through designing an appropriate fuzzy adaptive controller and combining the Lyapunov approach, Bouzeriba et al. discussed the projective synchronization in [39]. However, it is necessary to point out that constructing Lyapunov functions directly is not easy. In [40,41], Li et al. presented a graph-theoretic method to construct a global Lyapunov function for a system easily, which conquered the difficulty before.

Motivated by the above discussions, firstly, we take stochastic disturbances and time-varying coupling structure into coupled systems to serve the practical application better. Secondly, inspired by Mao et al., we apply a new type of feedback control based on discrete-time state observations on the response system to achieve synchronization. By means of the graph-theoretic method presented in [40,41], we construct a suitable Lyapunov function successfully. Next, two sufficient criteria are proposed to guarantee the exponential synchronization of SCSTN. Finally, we discuss stochastic coupled oscillators and provide two numerical examples to demonstrate the practicability of theoretical results.

Compared with the previous results, this paper has following contributions:

- The model we study is novel, which considers both time-varying coupling structure and discrete-time state feedback control.
- By combining the Lyapunov method with Kirchhoff's Matrix Tree Theorem, two sufficient criteria are given, whose conditions reflect the influence of topological structure on the dynamic properties.
- The theoretical results are applied to stochastic coupled oscillators, which indicates the practicability of obtained results and the significant role of discrete-time state feedback control.

The remainder of this paper is organized as follows. We devote Section 2 to designing models and presenting some assumptions and definitions. Then two kinds of sufficient criteria are obtained in Section 3. In Section 4, theoretical results are applied to stochastic coupled oscillators. And two numerical examples are proposed in Section 5. Finally, the conclusion is given in Section 6 and the proof of theorems is presented in Appendix.

Notations. Define $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P})$ as a complete probability space with a filtration $\mathbb{F} = \{\mathcal{F}_t\}_{t\ge 0}$ satisfying the usual conditions, and let $B(t)$ be a one-dimensional Brownian motion defined on the space. The mathematical expectation with respect to the given probability measure $\mathbb{P}$ is denoted by $\mathbb{E}(\cdot)$. The superscript "T" expresses the transpose of a vector or a matrix. Define $\mathbb{I} = \{1, 2, \cdots, i\}, \mathbb{Z}^+ = \{1, 2, \cdots\}, \mathbb{R}^1_+ = [0, +\infty)$ and $m = \sum_{l=1}^{i} m_l$ for $m_l \in \mathbb{Z}^+$. And $C^{2,1}(\mathbb{R}^n \times \mathbb{R}^1_+; \mathbb{R}^1_+)$ represents the family of all non-negative functions $V(x, t)$ on $\mathbb{R}^n \times \mathbb{R}^1_+$ which are continuously twice differentiable in x and once in t.

2. Model formulation

In this section, some systems, assumptions and definitions about the mean-square exponential synchronization and node Lyapunov functions are presented as follows:

Firstly, let $(\mathcal{G}, A(t))$ denote a digraph $\mathcal{G}$ with weight matrix $A(t)$, where $A(t) = (a_{kh}(t))_{i\times i}$, and more concepts about graph theory can be seen in [40]. Next, we consider a stochastic coupled system on the digraph $\mathcal{G}$ with i $(i\ge 2)$ vertices:

$$\mathrm{d}x_k(t) = \left(f_k(x_k(t), t) + \sum_{h=1}^{i} a_{kh}(t) H_{kh}(x_k(t), x_h(t)) \right)\mathrm{d}t + g_k(x_k(t), t)\mathrm{d}B(t), \;\; k \in \mathbb{I}, \tag{1}$$

where $f_k, g_k : \mathbb{R}^{m_k} \times \mathbb{R}^1_+ \to \mathbb{R}^{m_k}$ are continuous functions for any $x_k \in \mathbb{R}^{m_k}$, $H_{kh} : \mathbb{R}^{m_k} \times \mathbb{R}^{m_h} \to \mathbb{R}^{m_k}$ represents coupling form and $a_{kh}(t)$ is differentiable and corresponds to coupling strength.

We treat system (1) as a drive system. A discrete-time state feedback control is added to system (1), then response system is obtained as follows:

$$\mathrm{d}y_k(t) = \left(f_k(y_k(t), t) + u_k(y_k(\delta_t) - x_k(\delta_t), t) + \sum_{h=1}^{i} a_{kh}(t) H_{kh}(y_k(t), y_h(t)) \right)\mathrm{d}t + g_k(y_k(t), t)\mathrm{d}B(t), \;\; k \in \mathbb{I}, \tag{2}$$

where $u_k : \mathbb{R}^{m_k} \times \mathbb{R}^1_+ \to \mathbb{R}^{m_k}$ is a continuous function and $\delta_t = [t/\tau_k]\tau_k$, $\tau_k > 0$ is the duration between two consecutive observations.

To make readers better understand the drive-response systems, a simple example is given in Fig. 1. Among which the left is a drive system and the right is a response system. We define the k-th error state vector $e_k = y_k - x_k$ between systems (1) and (2), then we get the following error system:

$$\mathrm{d}e_k(t) = \left(\tilde{f}_k(t) + u_k(y_k(\delta_t) - x_k(\delta_t), t) + \sum_{h=1}^{i} a_{kh}(t) \tilde{H}_{kh}(t) \right)\mathrm{d}t + \tilde{g}_k(t)\mathrm{d}B(t), \;\; k \in \mathbb{I}, \tag{3}$$

where $\tilde{H}_{kh}(t) = H_{kh}(y_k(t), y_h(t)) - H_{kh}(x_k(t), x_h(t))$, $\tilde{f}_k(t) = f_k(y_k(t), t) - f_k(x_k(t), t)$ and $\tilde{g}_k(t) = g_k(y_k(t), t) - g_k(x_k(t), t)$.

Next, throughout this paper, some assumptions are presented as follows:

(1) For any $k \in \mathbb{I}$, $u_k(\cdot, t)$ satisfies the global Lipschitz condition and $f_k(\cdot, t), g_k(\cdot, t)$ satisfy the local Lipschitz condition. The Lipschitz constants are υ_k, χ_k, β_k, respectively.

(2) There exists a positive constant μ_{kh}, such that

$$|H_{kh}(y_k, y_h) - H_{kh}(x_k, x_h)| \le \mu_{kh}(|e_k| + |e_h|),$$

for any x_k, $y_k \in \mathbb{R}^{m_k}$.

(3) Digraph $(\mathcal{G}, A(t))$ is strongly connected where $A(t) = (a_{kh}(t))_{i\times i}$. In addition, it satisfies that $a'_{kh}(t) \le 0$, $a_{kh}(t) \le M$ and $M > 0$.

(4) Let $u_k(\cdot, t), f_k(\cdot, t), g_k(\cdot, t)$ and $H_{kh}(\cdot, \cdot)$ satisfy the linear growth condition.

Furthermore, according to assumptions (1), (2), (4) and Theorem 3.1 of Chapter 2 in [42], it is obvious that system (3) has a unique solution $e(t) = \left(e_1^{\mathrm{T}}(t), e_2^{\mathrm{T}}(t), \ldots, e_i^{\mathrm{T}}(t)\right)^{\mathrm{T}}$ for any initial conditions. Moreover, suppose that $u_k(0, t) = 0$, then system (3) has a trivial solution $e(t) \equiv 0$.

Before giving theorems, we present some definitions as follows.

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