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Distributed event-triggered scheme for a convex optimization problem in multi-agent systems ☆

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ABSTRACT

In this paper, a distributed event-triggered algorithm was proposed to solve a convex quadratic optimization problem of multi-agent systems under undirected and connected topologies. The event-triggered condition of each agent just requires its own state value and the state values of its neighbors at the triggering time, and hence the continuous communication and calculation are not required. Moreover, the minimum event-triggered interval is bounded by the sampling time and the Zeno behavior is therefore naturally avoided. The result is also extended to the networks with undirected and switching topologies. Numerical simulations show the effectiveness of the proposed approach.

1. Introduction

Over the past decade, various distributed coordination problems of multi-agent systems (MASs) have been extensively investigated due to their applications in many areas, such as surveillance systems, cooperation of multi-robot systems, intelligent transportation systems, and distributed economic dispatch of macro-grid, etc [1–6]. As a fundamental problem of the distributed coordination research, distributed consensus problem that aims to guarantee that a group of agents achieve consensus by exchange information with their neighbors is a hot focus [7]. As an application of distributed consensus, distributed optimization problems such as economic dispatch problem, optimal portfolio problem, optimal clustering problem, and resource optimization allocation problem have attracted great attention.

The distributed optimization problem with/without constraints aims to apply distributed algorithms to solve the optimization problem with the objective function being the sum of each agent' local objective function. Motivated by the distributed cooperative algorithms of MASs, distributed optimization schemes have been extensively investigated and proposed [8–20]. The distributed optimization algorithms can be classified into two categories: continuous algorithms [11,12,17–19] and discrete iterative algorithms [8–10,13–16]. These algorithms require agents to communicate with their neighbors continuously or periodically. Since each agent of multi-agent systems is equipped with an embedded digital microprocessor, control algorithms for multi-agent systems are traditionally implemented in periodic sample control mode in practice. Due to the lack of microprocessor energy, computational resource and communication capability, event-triggered control may be more suitable for the multi-agent systems. Compared with the traditional time triggered control and periodic sample control, event-triggered control can reduce the burden of computation and save the energy of systems.

Recently, distributed event-triggered control strategies are investigated and applied to solve consensus problem of MASs. Based on the work in [21], a distributed event-triggered scheme is proposed to solve the first order consensus problem and the event depends on the ratio of the measurement error to the norm of the local state function [22]. Meanwhile, the results were extended to a self-triggered setup where it is not required to keep track of the state error. The work in [23] presented a distributed event-based control strategy to solve the multi-agent average consensus problem and each agent's measurement error in the event-triggered condition was bounded by a time-dependent threshold. However, these algorithms in [22,23] require agents in the system to communicate with their neighbors continuously, which

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increases the communication consumption of systems. The work in [24] uses a combinational measurement approach to design the triggered event for rendezvous problem of multi-agent systems, where the continuous measurement of neighbors' states is avoided. In [25], two event-triggered conditions with or without continuous communication between neighboring agents were presented to solve the consensus problem, where the control input for each agent is only triggered at its own event time instants. The work in [26] considered the event-triggered control problem of homogenous multi-agent systems with switching topology. In [27], a self-triggered algorithm was developed and continuous monitoring of measurement errors was avoided. The work in [28] also apply the event-triggered control scheme to solve the average consensus problem of multi-agent systems, where the agents communicated with its neighbors and updated the control input only at the event instants. Meanwhile, a self-triggered algorithm was developed to avoid the continuous self state monitoring. However, all these algorithms mentioned above are implemented in continuous mode and the Zeno behavior is a common problem which is required to be carefully dealt with. The work in [29] addressed the consensus problem of multi-agent systems by using a sampled-data event detector, where the minimum inter-event time was bounded by the sampling period and the Zeno behavior was well solved. However, each agent and its neighbors are required to exchange information at each sampling instants. Based on the work in [29], sampling-based self-triggered consensus algorithm was proposed in [30], where the periodic communication at each sampling instants was not required.

To the best of our knowledge, only a few works have discussed the distributed optimization problem via distributed event-triggered schemes [31,32]. However, these works consider the distributed optimization problems without constraints. Motivated by the works [29] and [30], we present a distributed event-triggered algorithm to solve the convex quadratic optimization problem with equality constraints. By the Lyapunov method, the proper event-triggered conditions are designed and some sufficient conditions are derived to guarantee the convergence of the algorithm to the optimal solutions. Moreover, the Zeno behavior is avoided. The novelties of this paper lie in the following three aspects:

(1) We have skillfully converted the convex quadratic optimization problem with equality constraint to a distributed weighted average consensus problem and solved it via a distributed event-triggered strategy. The continuous or periodical communication and computation for each agents are not required. Therefore, the proposed algorithm has the potential to further reduce the communication and calculation burden and save the energy.
(2) The minimum inter-event time is bounded by the sampling period and the Zeno behavior is avoided. Moreover, according to the stability condition derived in the paper, the bounds of the sampling period and other parameters can be set in a easy way. Meanwhile, the impacts of the parameters have been analyzed.
(3) The constrained quadratic optimization problem for multi-agent systems under switching topology is also analyzed.

The rest of this paper is organized as follows. In Section 2, some basic concepts for algebraic graph theory are introduced and the constrained quadratic optimization problem is formulated. The distributed event-triggered algorithms under the fixed/switching topology and the detailed stability analysis are given in Section 3. The simulation results are presented in Section 4. Section 5 gives a summary of this paper.

2. Preliminaries and problem formulation

2.1. Basic graph theory

The network of multi-agent system can be modeled as a graph $G=(V,E,A)$. $V=\{v_1,\dots,v_n\}$ is the set of nodes that denotes the agents. $E\in\{V\times V\}$ is the edge set. $(v_i,\ v_j)\in E$ indicates that node j can receive information from node i. $A\in R^{n\times n}$ with elements $a_{i,j}$ represents the adjacency matrix of graph G. If $(v_i,\ v_j)\in E$, $a_{ij}=1$, otherwise, $a_{ij}=0$. It is assumed that there is no self-loop in graph G, and then $a_{ii}=0$. The degree matrix D of graph G is $diag\{d_1,d_2,\dots,d_n\}$ with $d_i=\sum_{j=1}^{n}a_{ij}$. The Laplacian matrix $L=\left[l_{ij}\right]_{n\times n}$ of graph G is defined as $l_{ii}=\sum_{i\neq j}a_{ij}$ for on-diagonal elements and $l_{ij}=-a_{ij}$ for off-diagonal elements.

For an undirected graph G, $(v_i,\ v_j)\in E$ implies $(v_j,\ v_i)\in E$ and $a_{ij}=a_{ji}$. The adjacency matrix A and the Laplacian matrix L are symmetrical. v_i and v_j are connected if there has a path consisted of a sequence of edges of the form $(v_1,v_2),(v_2,v_3),\dots,$ in the graph. An undirected graph G is a connected graph if any pair of nodes in the graph are connected. For an undirected and connected graph, 0 is a simple eigenvalue of L. The largest eigenvalue $\lambda_n(L)$ of L is upper-bounded by $2d_{max}$, i.e., $\lambda_n(L)<2d_{max}$ where $d_{max}=\max\{d_i,i=1,2,\dots,n\}$ and $d_{max}\le n-1$.

2.2. Problem formulation

Definition 1 [33]**.** A convex optimization problem is to find some $x^*\in\chi$ such that

$$f(x^*)=\min\{f(x):x\in\chi\}$$

for a convex constraint $\chi\in R^n$ and a convex objective function $f(x)$: $R^n\to R$.

The convex optimization problem investigated in this paper is stated as (2.1) and (2.2).

$$\underset{x_1,\dots,x_n}{\text{minimize}}\sum_{i=1}^{n}C_i(x_i(t)) \tag{2.1}$$

$$\text{subject to}\quad\sum_{i=1}^{n}x_i(t)=X_D, \tag{2.2}$$

where

$$C_i(x_i(t))=\alpha_i x_i^2(t)+\beta_i x_i(t)+\varphi_i \tag{2.3}$$

for $x_i(t)\ge 0$ and $\alpha_i,\ \beta_i,\ \varphi_i$ are the coefficients of the quadratic function $C_i(x_i(t))$. The coefficient α_i satisfies $\alpha_i>0$ such that the function $C_i(x_i(t))$ is a convex function. The value X_D is a constant and provides a constrain to the sum of $\sum_{i=1}^{n}x_i(t)$. Therefore, $\sum_{i=1}^{n}C_i(x_i(t))$ is a convex function with respect to $x_1(t),\dots,x_n(t)$. The convex quadratic optimization problem (2.1) with equality constraint (2.2) appears in the economic dispatch, the resources allocation, the optimal portfolio, and so on.

The problem (2.1) and (2.2) can be solved by the conventional centralized methods. The centralized methods need the information of all nodes and are not suitable for multi-agent systems. One of the purposes of this work is to provide a distributed solution. Based on the Lagrange multiplier method, one has that the solution set of (2.1) and (2.2) is equivalent to the solution set of (2.4).

$$\frac{\partial C_1(x_1(t))}{\partial x_1(t)}=\frac{\partial C_2(x_2(t))}{\partial x_2(t)}=\dots=\frac{\partial C_n(x_n(t))}{\partial x_n(t)}=\eta^*. \tag{2.4}$$

where η^* is the optimal Lagrangian multiplier.

In light of (2.3), (2.4) is equivalent to

$$2\alpha_1x_1(t)+\beta_1=2\alpha_2x_2(t)+\beta_2=\dots=2\alpha_nx_n(t)+\beta_n=\eta^*. \tag{2.5}$$