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Iterative projection based sparse reconstruction for face recognition☆

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ABSTRACT

This paper presents a projection based iterative method (PIM) for solving the L_1-minimization problem with its application to sparse representation and reconstruction. First, the unconstrained basis pursuit denoising (BPDN) problem is transformed into the cross-and-bouquet (CAB) form with a variable λ, and an iterative algorithm is proposed based on the projection method with the gradient of $\|x\|_1$ being transformed into a piecewise-linear function, which enhances the convergence of the algorithm. The global convergence of the algorithm is proved by Lyapunov method. Then, experiments conducted on random Gaussian sparse signals reconstruction and five well-known face data sets present the effectiveness and robustness of the proposed algorithm. It is also shown that the algorithm is robust to different sparsity levels and amplitude of signals, and has higher convergence rate and recognition accuracy compared with other L_1-minimization algorithms especially in the case of noise interference.

1. Introduction

In recent years, sparse representation is an important research topic in computer vision, especially on face recognition [1,2]. In the theory of sparse representation, test signal is represented by the sparse combination of the known signals, which can reduce the computational complexity. It is named as sparse representation based classification (SRC), which has been wildly used in many real applications.

In the literature, the sparse representation problem is always transformed into nonlinear optimization, especially L_0 and L_1-minimization problems. Many classic algorithms have been proposed for the minimization problems. An interior point algorithm is proposed in [3], which main idea is to transform the inequality constrained problem into an equality constrained problem and solve it by Newton method. To solve the basis pursuit problem, the Homotopy method [4] is specifically designed with least absolute shrinkage and selection operator. Methods based on local linear approximation, such as proximal-point method [5], reduced the computational complexity greatly. Also Lagrange dual method [6], and gradient projection method [7] have been proved efficient for solving the optimization problem.

More recently, some methods inspired by neural networks have been put up to solve the sparse representation problem. In [8–10], convolution neural network (CNN) has been applied to face recognition and proved effective in the experiments. Meanwhile, some recurrent neural networks based methods also have been applied in convex optimization. In [11,12], continuous-time recurrent neural networks have been used for linear programming problem. In [13,14], discrete-time recurrent neural networks are proposed for quadratic programming, and in [15,16], projection neural networks have been applied to solve variational inequalities and related optimization problems. The dynamical behavior and the convergency of the neural networks have been analyzed, and could be used for solving the sparse reconstruction problem.

In this paper, the proposed projection based iterative method is to solve the L_1-minimization problem based on CAB model that is equivalent to the unconstrained basis pursuit denoising (BPDN) problem. The method is based on projection operators, transformed the L_1-norm gradient into piecewise-linear function to enhance the convergence of the algorithm, and the global convergence is studied and proved by constructing the Lyapunov function.

The proposed method is applied for real applications, such as sparse signal reconstruction, face recognition with and without occlusion. The experimental results illustrate the effectiveness and robustness of the algorithm compared with the traditional SRC methods, the primal augmented Lagrangian method (PALM) which solves the original L_1-norm problem, truncated Newton interior-point method (TNIPM/L1LS) and fast iterative soft-thresholding algorithm (FISTA) which solve the L_1-norm problem with constrained condition. Furthermore, we compare the proposed algorithm with two state-of-the-art approaches based on robust regression models (RRM), the robust sparse coding (RSC) [17] and

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regularized robust coding (RRC) [18]. In the experiments, we combine the proposed algorithm with deep learning methods to improve the classification accuracy. The Deep Convolution Neural Network (DCNN) model named VGG-face [19] are utilized to extract the image features for training and testing, and then the proposed algorithm is utilized for classification.

The paper is organized as follows. In Section 2, the related work of the spare representation problem is described. In Section 3, the BPDN problem is transformed into a new form, and the projection operator based iterative method is proposed and analyzed. Experiments on signal reconstruction and face recognition are conducted in Section 4. Finally, Section 5 concludes the paper.

2. Related work

The constrained optimization has been widely used to approximate sparse signals in compressive sensing theory. We assume an unknown signal $x \in \mathbb{R}^n$ which is sparse, a test vector $b \in \mathbb{R}^m$, and a dictionary matrix $A \in \mathbb{R}^{m\times n}$, satisfying the constrained condition $b = Ax$. The main purpose of the sparse representation can be regarded as an inverse problem, obtaining x with the known A and b, which can be calculated by the L_0-minimization problem as following:

$$\begin{aligned} &\text{minimize} \quad \|x\|_0, \\ &\text{subject to} \quad Ax = b, \end{aligned} \tag{1}$$

where $\|\cdot\|_0$ is the L_0-norm.

The optimization problem in (1) is a NP-hard problem. According to the Restricted Isometry Property (R.I.P) condition, if the solution to the L_0-minimization problem is sparse enough, it is equivalent to the following L_1-minimization problem:

$$\begin{aligned} &\text{minimize} \quad \|x\|_1, \\ &\text{subject to} \quad Ax = b, \end{aligned} \tag{2}$$

where $\|\cdot\|_1$ is the L_1-norm.

Another form of the L_1-minimization problem is called the unconstrained basis pursuit denoising (BPDN) problem with a scalar weight λ:

$$\text{minimize} \quad \frac{1}{2}\|Ax - b\|_2^2 + \lambda\|x\|_1. \tag{3}$$

In this unconstrained problem, the noise is described by the L_2-norm; i.e., $\|Ax - b\|_2$.

Considering the situation where the test vector b is with an additional noise term e, there is another model called the cross-and-bouquet (CAB) model proposed in [20] as following

$$\begin{aligned} &\text{minimize} \quad \|z\|_1, \\ &\text{subject to} \quad [A, I]z = b, \end{aligned} \tag{4}$$

where $I \in \mathbb{R}^{n\times n}$ is an identity matrix and $z = [x^T, e^T]^T \in \mathbb{R}^{m+n}$. In the new dictionary $[A,\ I]$, the columns are highly correlated. The training vectors are tightly bundled as a "bouquet", and the vectors in the identity matrix and their negative counterparts form a "cross".

3. Projection based iterative method

Considering the noise that is described by the L_1-norm in (3) and the corresponding optimization problem is as follows

$$\text{minimize} \quad \|Ax - b\|_1 + \lambda\|x\|_1. \tag{5}$$

Let $\lambda y = b - Ax$, then problem (5) can be equivalently written as

$$\begin{aligned} &\text{minimize} \quad \lambda\|x\|_1 + \lambda\|y\|_1, \\ &\text{subject to} \quad Ax + \lambda y = b. \end{aligned} \tag{6}$$

Let $z = (x^T, y^T)^T$ and $B = (A\ \lambda I)$, then problem (6) can be further written as the CAB problem

$$\begin{aligned} &\text{minimize} \quad \lambda\|z\|_1, \\ &\text{subject to} \quad Bz = b. \end{aligned} \tag{7}$$

3.1. Projection operator

To solve the problem (7), we define the projection operator $g(u)$ (from $\mathbb{R}^n$ to $\Omega \subseteq \mathbb{R}^n$) as

$$g(u) = \arg\min_{v\in\Omega} \|v - u\|_2,$$

where $\|\cdot\|_2$ is the L_2-norm.

Here $\Omega = \{u \in \mathbb{R}^n : -1 \le u_i \le 1, i = 1, 2, \ldots, n\}$ is a hyper-rectangular set, and we have

$$g(u_i) = \begin{cases} 1, & u_i > 1, \\ u_i, & -1 \le u_i \le 1, \\ -1, & u_i < -1, \end{cases}$$

which is a piecewise-linear function.

Lemma 1 [21]**.** *For the projection operator $g(\cdot)$, the following inequality is true*

$$(h - v)^T\big(g(h) - g(v)\big) \ge \|g(h) - g(v)\|_2^2 \quad \forall\ h, v \in \mathbb{R}^n.$$

3.2. Method description

According to the Karush–Kuhn–Tucker (KKT) conditions [22], $z^* \in \mathbb{R}^{m+n}$ is an optimal solution to problem (7) if and only if there exist $\gamma^* \in \partial(\|z^*\|_1)$ and $w^* \in \mathbb{R}^m$ satisfying

$$\lambda\gamma^* + B^T w^* = 0, \tag{8}$$

$$Bz^* = b, \tag{9}$$

where $\partial(\|z^*\|_1)$ is the sub-differential of $\|z\|_1$ at z^*, and the ith component is defined as

$$\partial_i(\|z^*\|_1) = \begin{cases} 1, & z_i^* > 0, \\ [-1, 1], & z_i^* = 0, \\ -1, & z_i^* < 0. \end{cases} \tag{10}$$

Then, we have

$$\gamma^* = g(\gamma^* + z^*).$$

Let $u^* = \gamma^* + z^*$, then $\gamma^* = g(u^*)$ and $z^* = u^* - g(u^*)$. If the both sides of Eq. (8) are multiplied by B, it implies

$$w^* = -\lambda(BB^T)^{-1}Bg(u^*). \tag{11}$$

Substituting (11) into (8) results in

$$(I - P)g(u^*) = 0, \tag{12}$$

where $P = B^T(BB^T)^{-1}B$, and has the following property: $P^2 = P$.

According to (9), there is $Pz^* = q$ which is equivalent to $P(u^* - g(u^*)) = q$, where $q = B^T(BB^T)^{-1}b$. Combining with (12), we have

$$P(u^* - g(u^*)) + (I - P)g(u^*) - q = 0, \tag{13}$$

where u^* is the equilibrium point, and its corresponding value z^* is the optimal solution of problem (7).

Taking (13) as the decent value, the sparse reconstruction algorithm is written as:

$$u_{k+1} = (I - P)u_k - (I - 2P)g(u_k) + q, \tag{14a}$$

$$z_k = u_k - g(u_k). \tag{14b}$$

According to (14), the Algorithm 1 for solving the problem (7) is stated as following.