Neurocomputing 000 (2018) 1–7

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Finite-time consensus of multi-agent systems with continuous time-varying interaction topology

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ARTICLE INFO

Article history:
Received 10 January 2017
Revised 22 November 2017
Accepted 11 January 2018
Available online xxx

Communicated by Jiahu Qin

Keywords:
Finite-time consensus
Multi-agent systems
Continuous time-varying topology
Switching topology

ABSTRACT

This study investigates the finite-time consensus problem of multi-agent systems with continuous time-varying interaction topology. A distributed consensus protocol is presented to make the multi-agent system locally or globally reach consensus in finite time. The protocol is also used to analyze finite-time consensus of the multi-agent systems with fixed topology or modified switching topology. Numerical simulations are presented to illustrate the effectiveness of the obtained results.

1. Introduction

The distributed control of multi-agent systems has recently gained increasing interest owing to its broad applications to biology, physics, robotics, and control engineering. Typical examples include consensus problem [1], tracking control [2], formation control [3], flocking and swarming [4]. As one of the most typical collective behaviors in distributed control, the consensus problem is studied in this research.

One of the main objectives in the research on consensus problems is to design appropriate protocol or algorithm to guarantee the consensus of multi-agent systems. Studies have been conducted about both single-integrator dynamics and double-integrator dynamics. On the one hand, in view of first-order system, linear and nonlinear consensus protocols for the networks of dynamic agents with communication time-delays are introduced by Olfati-Saber and Murray [5]. They also discuss the consensus problems for networks of dynamic agents in more cases including directed networks with fixed topology or switching topology, and undirected networks with communication time-delays and fixed topology [1]. Other protocols are proposed for nonlinear stochastic dynamical networks [6,7], fractional-order systems [8,9], discrete-time multi-agent systems [10–13], impulsive systems [14] and so on. On the other hand, considering many practical multi-agent models, consensus for second-order dynamics has also been studied from various perspectives [15–22].

The above-mentioned consensus protocols of multi-agent systems are asymptotic consensus protocols, that is, their convergence time are infinite [5–22]. To improve the convergence rate, finite-time consensus which means the states of the agents can reach consensus in finite time, has drawn considerable attentions of researchers. Compared with consensus protocols, finite-time consensus protocols are more desirable, not only because of their faster convergence, but also their better performance in the presence of disturbances and uncertainties. Various finite-time consensus protocols have been proposed based on the theory of finite-time stability [23]. Cortés provides a class of discontinuous algorithms for the finite-time consensus of multi-agent systems with time-invariant or switching topology [24]. For discrete-time systems, Sundaram and Hadjicostis give a finite-time consensus algorithm that is built on the global information of interaction topology [25]. Wang and Xiao propose two finite-time consensus protocols for multi-agent systems with dynamic agents and address finite-time consensus problems under the conditions of time-invariant topology or switching undirected topology [26]. Wang and Hong construct several distributed finite-time consensus rules for multi-agent dynamics and consider finite-time χ-consensus problem for a multi-agent system with first-order individual dynamics and switching interaction topologies [27]. Zheng and Wang investigate finite-time consensus of heterogeneous multi-agent systems [28]. Liu et al. design a switching consensus protocol such that the finite-time

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https://doi.org/10.1016/j.neucom.2018.01.004

consensus on a fixed directed interaction graph can be achieved in faster speed [29].

The present paper focuses on the design and analysis of distributed finite-time consensus protocols for multi-agent systems over fixed and switching topologies. In practice, on the one hand, the topology of multi-agent systems is complex, may not necessarily be fixed or switching. And on the other hand, the movement of the agents may lead the graph topology changing with time. Therefore, it is important to consider multi-agent systems with continuous time-varying interaction topology. However, few studies have been conducted on these systems, and these few results are about consensus. And there are no results about finite-time consensus of multi-agent systems with continuous time-varying topology. To fill this vacancy, this paper proposes a protocol for this multi-agent systems. The protocol in this paper is designed in a general form and ensures that the multi-agent systems possesses continuous time-varying undirected interaction topology. The protocol is expected to contain types of practical distributed protocols as its special cases, such as the protocols for the multi-agent systems with fixed topology or switching topology.

This paper is organized as follows. Section 2 presents the useful notions and results about graph theory and the lemma for finite-time convergence analysis. Section 3 formulates the finite-time consensus problem and designs the protocol. Section 4 studies the finite-time consensus problem under the protocol and applies the obtained results to the multi-agent system with fixed topology or modified switching topology. Section 5 gives two simulations.

2. Preliminaries

In this paper, a weighted undirected graph will be used to model the undirected interaction topology among agents.

In general, an undirected graph $\mathcal{G}$ consists of a vertex set $\mathcal{V}(\mathcal{G})=\{v_1,v_2,\dots,v_n\}$ and an edge set $\mathcal{E}(\mathcal{G})\subseteq\{(v_i,v_j):v_i,v_j\in\mathcal{V}(\mathcal{G})\}$, where (v_i, v_j) denotes an unordered pair of vertices v_i, v_j. A weighted undirected graph $\mathcal{G}(A)$ is an undirected graph $\mathcal{G}$ with a non-negative and symmetric weight matrix $A=(a_{ij})\in\mathbb{R}^{n\times n}$, such that $(v_i,v_j)\in\mathcal{E}(\mathcal{G})$ if and only if $a_{ij}=a_{ji}>0$, i.e., $a_{ij}=a_{ji}>0$ is the weight of edge $(v_i,v_j)\in\mathcal{E}(\mathcal{G})$ and $a_{ij}=a_{ji}=0$ implies that $(v_i,v_j)\notin\mathcal{E}(\mathcal{G})$.

An undirected graph $\mathcal{G}_s$ is a subgraph of $\mathcal{G}$ if $\mathcal{V}(\mathcal{G}_s)\subseteq\mathcal{V}(\mathcal{G})$ and $\mathcal{E}(\mathcal{G}_s)\subseteq\mathcal{E}(\mathcal{G})$. A subgraph $\mathcal{G}_s$ with $\mathcal{V}(\mathcal{G}_s)=\mathcal{V}(\mathcal{G})$ is called a spanning subgraph of $\mathcal{G}$. In $\mathcal{G}$, a path from v_{i_0} to v_{i_k} is a sequence $v_{i_0},v_{i_1},\dots,v_{i_k}$ of finite distinct vertices such that $(v_{i_j},v_{i_{j+1}})\in\mathcal{E}(\mathcal{G})$ for all $j=0,1,2,\dots,k-1$. If a sequence $v_{i_0},v_{i_1},\dots,v_{i_k}$ of finite distinct vertices is a path in $\mathcal{G}$ from v_{i_0} to v_{i_k} denoted by P and $(v_{i_k},v_{i_0})\in\mathcal{E}(\mathcal{G})$ is different from all the edges in P, then the sequence $v_{i_0},v_{i_1},\dots,v_{i_k},v_{i_0}$ of finite vertices is called a cycle in $\mathcal{G}$. If there is a path from any vertex to any other one in $\mathcal{G}$, then $\mathcal{G}$ is connected. A tree is a connected undirected graph that contains no cycles. A spanning tree of $\mathcal{G}$ is a spanning subgraph that is a tree.

Following result about the undirected graph is important to our main results.

Lemma 1 [30]**.** *Suppose that $\mathcal{G}$ is an undirected graph. Then the following results hold.*

(i) *If $\mathcal{G}$ is connected, then $\mathcal{G}$ has a spanning tree.*
(ii) *If $\mathcal{G}$ is a tree, then $|\mathcal{E}(\mathcal{G})|=|\mathcal{V}(\mathcal{G})|-1$ and there is a unique path from any vertex to any other one in $\mathcal{G}$.*

Next, another lemma is given for finite-time convergence analysis.

Consider the following p-dimensional system:

$$\dot{x}(t)=f(t,x(t)),\tag{1}$$

where $f:[0,+\infty)\times\mathbb{R}^p\to\mathbb{R}^p$ is continuous on its domain and $f(t,0)\equiv0$. According to the *Existence Theorem of Peano*, there exists a sufficiently small positive number τ and a solution $x:(-\tau,\tau)\to\mathbb{R}^p$ of (1) such that $x(0)=x_0$ for each $x_0\in\mathbb{R}^p$, and every solution of (1) has an extension that is right maximally defined.

Suppose $V:[0,+\infty)\times\mathbb{R}^p\to\mathbb{R}$ is a continuous function. If V is continuously differentiable on its domain, then the derivative of V is defined along the solutions of (1) as [31, p. 324]

$$\dot{V}\Big|_{(1)}(t,x)=\frac{\partial V(t,x)}{\partial t}+\frac{\partial V(t,x)}{\partial x}f(t,x).\tag{2}$$

Let $r:\mathbb{R}\to\mathbb{R}$ be a continuous function, then $r\in\mathcal{R}$ if the following two statements hold.

(A1) $r(0)=0$.
(A2) when $x\neq0$, $xr(x)<0$ and $\int_x^0\frac{1}{r(u)}\mathrm{d}u<+\infty$.

Lemma 2. *Assume that there exist three continuous functions $V:[0,+\infty)\times\mathbb{R}^p\to\mathbb{R}$, $c:[0,+\infty)\to\mathbb{R}$ and $r:\mathbb{R}\to\mathbb{R}$ such that the following three statements hold.*

(B1) *V is continuously differentiable satisfying that $V(t,0)\equiv0$, $V(t,x)>0$ for all $x\neq0$, and $V(t,x)\to+\infty$ as $\|x\|\to+\infty$.*
(B2) *c is non-negative and $r\in\mathcal{R}$.*
(B3) $\dot{V}\Big|_{(1)}(t,x)\le c(t)r(V(t,x)),\ \ (t,x)\in[0,+\infty)\times\mathbb{R}^p$.

Then for every $x_0\in\mathbb{R}^p$, any right maximally solution $x(t)$ of (1) with $x(0)=x_0$, it is established that

(i) *if there exists $T>0$ such that $\int_0^T c(\tau)\mathrm{d}\tau>0$, for any x_0 when $\int_{V(0,x_0)}^0\frac{\mathrm{d}u}{r(u)}\le\int_0^T c(\tau)\mathrm{d}\tau$, $x(t)=0$ for all t that satisfies $\int_0^t c(\tau)\mathrm{d}\tau\ge\int_{V(0,x_0)}^0\frac{\mathrm{d}u}{r(u)}$, then $x(t)=0$ is local finite-time stable.*
(ii) *if $\int_0^{+\infty}c(\tau)\mathrm{d}\tau=+\infty$, $x(t)=0$ for all t that satisfies $\int_0^t c(\tau)\mathrm{d}\tau\ge\int_{V(0,x_0)}^0\frac{\mathrm{d}u}{r(u)}$, then $x(t)=0$ is global finite-time stable.*

Proof. Suppose that $x_0\in\mathbb{R}^p$ and any right maximally defined solution $x(t)$ of (1) satisfying $x(0)=x_0$ is defined on $[0,\tau_{x_0})$. It can be shown that $\dot{V}\Big|_{(1)}(t,x)\le0$. This implies that $V(t,x(t))\le V(0,x_0)$ for all $t\in[0,\tau_{x_0})$. If $\tau_{x_0}<+\infty$, then by the Extension Theorem of Solution, $\|x(t)\|\to+\infty$ as $t\to\tau_{x_0}^-$, and moreover, $V(t,x(t))\to+\infty$ as $t\to\tau_{x_0}^-$, this is contradiction to $V(t,x(t))\le V(0,x_0)$ for all $t\in[0,\tau_{x_0})$. Thus, $x(t)$ is defined on $[0,+\infty)$. By formula (2) and statement (B3),

$$\frac{\mathrm{d}V(t,x(t))}{\mathrm{d}t}\le c(t)r(V(t,x(t))),\ \ t\in[0,+\infty).$$

If $x_0=0$, then $x(t)\equiv0$. Suppose that $x_0\neq0$ and $\int_{V(0,x_0)}^0\frac{\mathrm{d}u}{r(u)}\le\int_0^T c(\tau)\mathrm{d}\tau$. If $x(t)\neq0$ for all $t\in[0,+\infty)$, then

$$\int_{V(0,x_0)}^0\frac{\mathrm{d}u}{r(u)}>\int_{V(0,x_0)}^{V(t,x(t))}\frac{\mathrm{d}u}{r(u)}=\int_0^t\frac{\mathrm{d}V(\tau,x(\tau))}{r(V(\tau,x(\tau)))}\ge\int_0^t c(\tau)\mathrm{d}\tau,$$

for all $t\in[0,+\infty)$, and moreover, $\int_{V(0,x_0)}^0\frac{\mathrm{d}u}{r(u)}>\int_0^T c(\tau)\mathrm{d}\tau$. This is a contradiction. Thus, there exists $t\in[0,+\infty)$ such that $x(t)=0$. Let

$\hat{t}=\inf\{t\in[0,+\infty):x(t)=0\}$.

Then $\hat{t}>0$, $x(\hat{t})=0$ and $x(t)\neq0$ for all $t\in[0,\hat{t})$. It is easy to obtain that $x(t)=0$ for all $t\ge\hat{t}$, and $\int_{V(0,x_0)}^0\frac{\mathrm{d}u}{r(u)}\ge\int_0^{\hat{t}}c(\tau)\mathrm{d}\tau$. If $\int_0^t c(\tau)\mathrm{d}\tau\ge\int_{V(0,x_0)}^0\frac{\mathrm{d}u}{r(u)}$, then $t\ge\hat{t}$ and thus, $x(t)=0$. This prove (i).

(ii) can be proved in the same way. □

Lemma 2 provides an method to solve the problem of finite-time stable of non-autonomous systems which is different from the commonly finite-time stable theorem. The results of Lemma 2 is more general, and the results may be more relaxed. Since our main work is to study finite-time consensus problem of multi-agent system with continuous time-varying interaction topology, we do not elaborate on it.