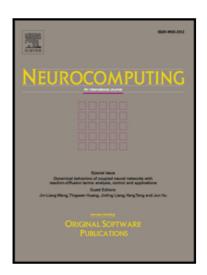# Accepted Manuscript

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Linear-Quadratic Optimal Control for Unknown Mean-Field Stochastic Discrete-Time System via Adaptive Dynamic Programming Approach

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Abstract: We develop an adaptive dynamic programming (ADP) approach to deal with the linear-quadratic (LQ) optimal control problem with unknown discrete-time mean-field stochastic system in this paper. At first, the mean-field stochastic LQ problem is transformed into the deterministic case by system transition. Secondly, a value iteration ADP approach is proposed and convergence proof is also discussed. Once again, in order to achieve the iteration method without any knowledge of the dynamics, back propagation neural network (BPNN) is applied to design model network, critic network and action network to ensure unknown system model, value function and control strategy, respectively. At last, it is demonstrated that the ADP approach is valid through simulation.

Keywords: Linear-quadratic optimal control; Mean-field stochastic system; Adaptive dynamic programming; Back propagation neural network

1   Introduction

During the past decades, the control and nalysis problems of stochastic systems have attracted a great of interest, for example, H_2/H_∞ problems [1–3], adaptive fuzzy control issues [4–6], event-based control research [7, 8], distributed recursive filter problem [9] and epidemic modeling studies [10–12] and so on [13, 14]. It is notable that several works have been published on LQ problem, such as discrete-time situations [13, 15, 16], and continuous-time situation [17], as well as with Markov jumps [17–19] and infinite horizon case [20]. The LQ optimal control problem for mean-field stochastic system was investigated in [21–23], which has been widely used in various areas, such as games, economics, engineering and so on. Unlike the traditional stochastic LQ control issues, mean-field type includes the value function and the system both contain the states and the controls along with their expected values, in other words, mean-field scheme is used to research the collective acts arising from the interaction of the individual in all kinds of physical and sociological systems. At present, it has gained considerable attention from the mathematics to control communities [23] and stochastic maximum principles of this scheme are obtained comprehensively research in various works [24–27]. In the aspect of application, the LQ optimal control problems have arisen naturally in lots of practical cases. As such, it makes great practical significance to study the LQ optimal control problem for mean-field stochastic system.

It is valuable to mention that the well-posedness of the LQ problem and the solvability of generalized difference Riccati equations (GDREs) are equivalent, see [21, 22]. For simple LQ case, it is more easier to obtain an optimal control via the solution of GDREs by the kernel-range decomposition representation of the expectation operator and its pseudo-inverse, see [22, 28, 29]. However, the approaches referred to above are all based on the precondition that

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