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Brief papers

# Stability analysis of inertial Cohen–Grossberg neural networks with Markovian jumping parameters[☆]

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ABSTRACT

In this paper, we concentrate on the problem of global asymptotical stability for a class of Markovian jump inertial Cohen–Grossberg neural networks. The jumping parameters are described with a continuous-time, finite-state Markov chain. By adopting the method of model transformation, differential mean value theorem, Lyapunov stability theory and linear matrix inequality techniques, we derive some novel sufficient conditions to guarantee the global asymptotical stability for the addressed systems. It is worth mentioning that the model investigated in this letter comprises and generalizes many existing results in the previous literature. Finally, the effectiveness of the theoretical results is validated by numerical examples.



## 1. Introduction

In last decades, neural networks have received widespread interests in researchers due to their wide applications in various fields such as pattern recognition, associative memory, parallel computing and image processing [1–6]. Among them, the Cohen–Grossberg neural network (CGNN), which was first proposed by Cohen–Grossberg [7], is the most representative one since it comprises many popular neural networks as its special cases, such as Hopfield neural networks and cellular neural networks. It is rather general and can describe a number of models arising from neurobiology, population biology and evolution theory [8], which motivates the investigation on the stability analysis of CGNNs [9,10]. On the other hand, time delays are often unavoidable in real situations, which may lead to oscillation and instability of neural networks. Several typical examples of delayed CGNNs can be found in chemical processes, population dynamics and even aircraft systems. Therefore, the dynamical behaviors of CGNNs with time delays have been paid much considerable attention and many interesting results have been proposed, see [11–16] and references therein.

It is worth pointing out that the majority of the existing literature concentrate on the neural networks with only first derivative of the states, while the influence made by the inertia item is often ignored. The introduction of the inertia item can be regarded as a powerful tool to generate complicated bifurcation behavior and chaos in a networked system. For example, Li et al. [17] observed an obvious chaotic behavior by adding the inertia term to a delayed differential equation; the authors in [18] added the inertia term to the Hopfield effective-neuron system and also found chaos phenomenon; Babcock and Westervelt [19] pointed out that the dynamics could be complex when the neuron couplings included an inertial nature. Recently, some research results about dynamic behaviors of inertial neural networks have come out. For instance, Cao and Wan [20] adopted the matrix measure strategies to investigate the stability of inertial BAM neural network; the pinning synchronization problem of coupled inertial delayed neural network was investigated in [21]; Tu et al. [22] studied the global exponential stability for inertial neural networks. In addition, some global exponential stability conditions for inertial CGNNs with time delays were proposed in [23,24].

On the other hand, systems with Markovian jumping parameters have been widely utilized to model a number of practical systems where they may undergo abrupt changes in their structure and parameters. In other words, the neural networks may have finite modes and the modes may switch from one to another at

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different times. The switching between different modes can be governed by continuous-time Markov chain. Such kind of neural network is of great significance and has received enough attention. For example, Wang et al. [25] and Zhang and Wang [26] investigated the stability of stochastic CGNNs with Markovian switching; Dong et al. [27] combined the effect of impulse with Markovian jumping CGNNs with mixed time delays; some other interesting results concerning the dynamic behaviors of complex systems with Markovian jumping parameters could be found in [28-32]. However, to the best of our knowledge, there have been few results on the stability of inertial Cohen–Grossberg neural networks with Markovian jumping parameters, which motivates the present research.

Based on the above discussions, this paper is intended to investigate the global asymptotical stability of inertial Cohen–Grossberg neural networks with Markovian jumping parameters. By employing the Lyapunov functional as well as linear matrix inequality techniques, several novel sufficient conditions are provided to guarantee the global asymptotical stability of the equilibrium point for the proposed system. Compared with the existing literature, the contributions of this paper are mainly displayed in three aspects: (i) The proposed model is quite general since many factors such as Markovian jumping parameters and time-varying delays are considered. (ii) The construction of Lyapunov functional is rather special, where the use of delay-decomposition technique makes the sufficient conditions less conservative. (iii) All the obtained results are independent of the derivative of time-varying delays, which indicates that the restriction on the derivative of $\tau(t)$ is removed.

The remainder of this paper is organized as follows. In Section 2, a class of inertial Cohen–Grossberg Markovian jumping networks is formulated, and some essential assumptions and lemmas are introduced. Several sufficient conditions that guarantee the global asymptotical stability of the equilibrium point are given in Section 3. In Section 4, two numerical examples are proposed to verify the effectiveness of the derived results. Finally, we conclude this paper with several general remarks in Section 5.

## 2. Model description and preliminaries

### 2.1. Model description

Let $\{r(t),\ t \geq 0\}$ be a continuous-time Markovian process with right continuous trajectories and taking values in a finite set $S = \{1, 2, \ldots, N\}$ . The transition probability from mode $p$ at time $t$ to mode $q$ at time $t+\Delta t$ with generator $\Pi = (\pi_{pq})_{N\times N}$ is given by

$$P\{r(t+\Delta t) = q | r(t) = p\} = \begin{cases} \pi_{pq}\Delta t + o(\Delta t), & p \neq q, \\ 1 + \pi_{pq}\Delta t + o(\Delta t), & p = q, \end{cases}$$

where $\Delta t > 0$ and $\lim_{\Delta t\to 0} \frac{o(\Delta t)}{\Delta t} = 0$. Here, $\pi_{pq} \geq 0$ is the transition probability rate, and satisfies $\pi_{pp} = -\sum_{q=1,q\neq p}^{N} \pi_{pq}$.

In this paper, the inertial Cohen–Grossberg neural networks with Markovian jumping parameters can be described by the following differential equations:

$$\begin{aligned} \frac{d^2x_i(t)}{dt^2} = & -\delta_i \frac{dx_i(t)}{dt} - \mu_i(x_i(t), r(t)) \Bigg[ h_i(x_i(t), r(t)) \\ & - \sum_{j=1}^{n} a_{ij}(r(t)) g_j(x_j(t)) \\ & - \sum_{j=1}^{n} b_{ij}(r(t)) g_j(x_j(t-\tau(t))) + J_i \Bigg], \\ & t \geq 0, \quad i = 1, 2, \ldots, n, \end{aligned} \tag{1}$$

where $x_i(t)$ is the state variable of the $i$th neuron at time $t$, the second derivative of $x_i(t)$ is an inertial term, $\delta_i > 0$ are constants, $\mu_i(\cdot)$ represents an amplification function, and $h_i(\cdot)$ stands for an appropriately behaved function, $a_{ij}$ and $b_{ij}$ are the connection weights of the neural networks, the activation function $g_j(\cdot)$ describes the manner in which the neurons respond to each other, $J_i$ stands for the external input on the $i$th neuron, and $\tau(t)$ denotes the time-varying transmission delay satisfying $0 < \tau(t) \leq \tau$.

By introducing the following variable transformation

$$y_i(t) = \frac{dx_i(t)}{dt} + x_i(t), \qquad i = 1, 2, \ldots, n.$$

Then, system (1) can be written as

$$\begin{cases} \dfrac{dx_i(t)}{dt} = -x_i(t) + y_i(t), \\ \dfrac{dy_i(t)}{dt} = -(1-\delta_i)x_i(t) - (\delta_i - 1)y_i(t) \\ \qquad -\mu_i(x_i(t), r(t)) \Big[ h_i(x_i(t), r(t)) - \sum_{j=1}^{n} a_{ij}(r(t)) g_j(x_j(t)) \\ \qquad - \sum_{j=1}^{n} b_{ij}(r(t)) g_j(x_j(t-\tau(t))) + J_i \Big]. \end{cases} \tag{2}$$

For the sake of simplicity, for each $r(t) = p \in S$, set $\mu_i(x_i(t), r(t)) = \mu_{pi}(x_i(t))$, $h_i(x_i(t), r(t)) = h_{pi}(x_i(t))$, $a_{ij}(r(t)) = a_{pij}$, and $b_{ij}(r(t)) = b_{pij}$.

Then system (2) can be rewritten as

$$\begin{cases} \dfrac{dx_i(t)}{dt} = -x_i(t) + y_i(t), \\ \dfrac{dy_i(t)}{dt} = -(1-\delta_i)x_i(t) - (\delta_i - 1)y_i(t) - \mu_{pi}(x_i(t)) \Big[ h_{pi}(x_i(t)) \\ \qquad - \sum_{j=1}^{n} a_{pij} g_j(x_j(t)) - \sum_{j=1}^{n} b_{pij} g_j(x_j(t-\tau(t))) + J_i \Big]. \end{cases} \tag{3}$$

Throughout this paper, the following assumptions are utilized.

$(H_1)$: For $\forall v_1, v_2 \in \mathbb{R}$, $v_1 \neq v_2$, the activation functions $g_i(\cdot)$ comply with the following restriction:

$$0 \leq \frac{g_i(v_1) - g_i(v_2)}{v_1 - v_2} \leq l_i, \quad i = 1, 2, \ldots, n,$$

where $l_i > 0$ are constants. Define $L = \mathrm{diag}(l_1, l_2, \ldots, l_n)$.

$(H_2)$: The amplification functions $\mu_{pi}(\cdot)$ are continuous and bounded, satisfy

$$0 < \underline{\alpha}_{pi} \leq \mu_{pi}(\cdot) \leq \bar{\alpha}_{pi}.$$

Besides, define

$$\underline{\alpha}_p = \min_{1\leq i\leq n}(\underline{\alpha}_{pi}), \quad \bar{\alpha}_p = \max_{1\leq i\leq n}(\bar{\alpha}_{pi}).$$

$(H_3)$: For $i = 1, 2, \ldots, n$, there exist positive constants $\underline{\beta}_{pi} > 0$, $\bar{\beta}_{pi} > 0$, such that the behaved functions $h_{pi}(\cdot)$ are subjected to

$$0 < \underline{\beta}_{pi} \leq h'_{pi}(\cdot) \leq \bar{\beta}_{pi}.$$

Besides, set

$$\underline{\beta}_p = \min_{1\leq i\leq n}(\underline{\beta}_{pi}), \quad \bar{\beta}_p = \max_{1\leq i\leq n}(\bar{\beta}_{pi}).$$

It is well known that the bounded activation functions always guarantee the existence of an equilibrium $(x^*, y^*)^T$ for system (3). For convenience, we shift the equilibrium point to the origin by transformation

$$u_i(t) = x_i(t) - x_i^*, \quad v_i(t) = y_i(t) - y_i^*.$$

Then system (3) can be transformed into the following form: