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Global stability of complex-valued recurrent neural networks with both mixed time delays and impulsive effect

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ABSTRACT

The global stability problem for a class of complex-valued neural networks with both mixed time delays and impulsive effect is concerned in this literature. By using Schur complement, forming an equivalent matrix, and constructing an appropriate Lyapunov functional, some new sufficient criteria to ascertain the existence, uniqueness, globally exponential stability and globally asymptotical stability of the equilibrium point of complex-valued neural networks are obtained in terms of linear matrix inequality techniques. Meanwhile, several linear matrix inequalities have been proved in the literature, which shall be used to prove the stability of the equilibrium point. Finally, simulation results of one numerical example are also delineated to demonstrate the effectiveness of our theoretical results.

1. Introduction

During the past decades, recurrent neural networks have received more and more considerable attention and gotten constant development due to their wide applications in many main scientific research fields such as pattern classification, signal processing, associative memory design, control and optimization [1–11]. The instability and oscillation are always existence since various kinds of time delays appear in many systems. Therefore, the problem on the stability of a delayed recurrent neural networks has been a hot topic in a period of time for many explorers and reference therein (see [12–21]).

As an extension of real-valued neural networks, the states, connection weight matrices, activation functions, and external inputs of complex-valued neural networks are always defined in the complex domain and thus they have strongly desired properties due to their extensive applications in electromagnetic light, and quantum waves. For example, complex-valued neural networks make it possible to solve some more complex problems than their real-valued ones, such as the XOR problem and the detection of symmetry problem that can be solved by a single complex-valued neuron under the orthogonal decision boundaries, but they cannot be solved with their real-valued counterparts, which reveal the strengthen potent computational power of complex-valued neurons [16–18]. Therefore, it is necessary for us to study the dynamic behaviors of considered complex-valued neural networks that dig deeply into their stability analysis. Especially, they reflect a critical advantage in diverse fields of engineering along with the development of analytic signals, where signals are routinely analyzed in time. Many outstanding works have been done for complex-valued neural networks.

Through the previous studying on the dynamical behaviors of complex-valued neural networks, some explorers have proposed many appropriate approaches such as energy function approach, Lyapunov function approach and synthesis approach and so on (see [22–24]). In [24], by separating complex-valued neural networks into real and imaginary parts, forming an equivalent real-valued system, and constructing a Lyapunov functional, some sufficient conditions to guarantee existence, uniqueness and globally stability of the equilibrium point are investigated. In [25], the complete stability for a delayed complex-valued neural networks with impulsive effect is studied and several sufficient conditions to ascertain the complete stability of equilibrium points are derived by using the Lyapunov functional, stability theory and impulse technique. In fact, impulsive phenomena universally exist in all kinds of scientific fields involving physics, chemical technology and economics [26].

Moreover, because of the existence of plenty of parallel paths with several axon sizes, every neural network always has spatial properties in reality. The detail indicates that the signal propagation is not frequently being instantaneous in the neural networks. Thus, it is essential to introduce distributed delays in our systems to make our systems have more capacity, even closer to reality.

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Contributed to the inevitable appearance of time-varying delays in practical applications, quite recently, the problems for the global stability of a delayed complex-valued neural networks with two assumptions of complex-valued activation functions are considered in [23]. But Assumption 1 about activation functions is covered by Assumption 2 in [23], and the point makes the applications of the result quite limited in [23]. In the paper, inspired by [23], we study global stability of complex-valued recurrent neural networks with both mixed time delays and impulsive effect. Our requirements of activation functions are only based on Assumption 2 of [23], and the other constrained conditions are less restrictive than before literatures [23,24] and even vanished in our paper.

The remainder of this paper is organized as follows. In Section 2, we give some useful lemmas, definition, and the problem statement. We obtained some global stability results for complex-valued recurrent neural networks in Section 3. One numerical example is given in Section 4 to demonstrate the effectiveness of the theoretical results. Conclusions are given in Section 5.

Notations. The notations are quite standard. Throughout this paper, i shows the imaging unit, i.e., $i=\sqrt{-1}$. For a complex number $z=x+iy$, the notions $|z|=\sqrt{x^2+y^2}$, C and C^n denotes respectively, the set of all complex numbers and the set of all n-dimensional complex-valued vectors, N^+ denotes the set of positive integers. I_N denotes the identity matrix with dimension N. For a square matrix A, let A^* denotes its conjugate transpose and $\lambda_{\max}(A)(\lambda_{\min}(A))$ denotes the maximum (minimum) eigenvalue of a symmetric A. For a real symmetric matrix B, $B>0$ $(B<0)$ if B is the positive (negative) definite. $\|\cdot\|$ denotes the vector Euclidean norm.

2. Preliminaries

In this section, some basic concepts about the linear matrix inequality are illustrated. Moreover, we consider the following complex-valued neural networks with both mixed delays and impulsive effect.

$$\begin{cases} \dot{z}(t) = -Cz(t)+Af(z(t))+Bf(z(t-\tau(t))) \\ \qquad +Q\int_{t-\beta}^{t} f(z(s))ds+I(t), & t\neq t_k, \\ z(t) = P_k h_k(z(t^-))+E_k s_k(z((t-\tau(t))^-))+J_k(t), & t=t_k. \end{cases} \tag{1}$$

For $t\geq 0$, where $z(t)=(z_1(t),z_2(t),\dots,z_n(t))^T\in C^n$ denotes the state vector; $f(z(t))=(f_1(z_1(t)),\dots,f_n(z_n(t)))^T\in C^n$ represents the complex valued vector function; $\tau_{ij}(t)$ represents to the transmission delay along the axon of the ith unit and satisfies $0\leq\tau_{ij}(t)\leq\tau_{ij}(\tau_{ij}$ is a constant); $C=diag(c_1,\dots,c_n)\in R^n$ is the self-feedback connection weight matrix, where $c_i>0$; $A=(a_{ij})_{n\times n}\in C^{n\times n}$, $B=(b_{ij})_{n\times n}\in C^{n\times n}$ and $Q=(q_{ij})_{n\times n}\in C^{n\times n}$ are the connection weight matrices; $I(t)=(I_1(t),\dots,I_n(t))\in C^n$ and $J(t)=(J_1(t),\dots,J_n(t))\in C^n$ are the external input vector without impulsive effect and with impulsive effect at time t, respectively. The second part is discrete part of system (1), which depicts that the evolution processes experience abrupt change of state at the immediate of time t_k(called impulsive moments), where $z_j(t^-)$ and $z_j((t-\tau_{ij}(t))^-)$ delegate the left limit of $z_j(t)$ and $z_j(t-\tau_{ij}(t))$, respectively; $P_k(z_1(t^-),\dots,z_n(t^-))$ delegates impulsive perturbations of the state vector at time t_k, and $E_k(z_1(t-\tau_{i1})^-,\dots,z_n(t-\tau_{in})^-)$ represents impulsive perturbations of the state vector at time $t_k-\tau(t_k)$; the fixed moments of time t_k satisfy $0<t_1<t_2<\cdots$, $\lim_{k\longrightarrow+\infty}t_k=+\infty$.

The initial conditions of system (1) are in the form of $z_i(s)=\phi_i(s), s\in(-\infty,0]$, where ϕ_i is bounded and continuous on $(-\infty,0]$.

For a complex-valued recurrent neural networks, we have no choice but to pick out activation functions even if it is one of the fundamental challenges. Any regular analytic function cannot be bounded or else it reduces to a constant according to *Liouville's theorem*. In other words, activation functions in complex-valued neural networks cannot be both bounded and analytic. Throughout this literature, let $f_j(\cdot)$ be a set of complex-valued function, there is one class of complex-valued activation function satisfying the following assumptions.

Assumption 1. If $\bar{z}$ is an equilibrium point of the continuous part of system (1), then it has also the equilibrium point of the discrete part of system (1) and satisfies the following condition

$$\bar{z}=P_k h_k(\bar{z})+E_k s_k(\bar{z})+J_k,\ k=1,2,\dots. \tag{2}$$

Assumption 2. For the function $f(\cdot)$, there exists a positive matrix $F=diag(F_1,\dots,F_n)$ such that for any $u_1,\ u_2\in C;\ i=1,2,\dots$

$$|f_i(u_1)-f_i(u_2)|\leq F|u_1-u_2|. \tag{3}$$

Assumption 3. There exist positive matrices $H_k=diag(H_{k1},H_{k2},\dots,H_{kn})$ and $S_k=diag(S_{k1},S_{k2},\dots,S_{kn})$ such that

$$|h_{ki}(u_1)-h_{ki}(u_2)|\leq H_{ki}|u_1-u_2|,$$
$$|s_{ki}(u_1)-s_{ki}(u_2)|\leq S_{ki}|u_1-u_2|,$$

for all $u_1,\ u_2\in C;\ k=1,2,\dots$.

Remark 1. Actually, as an extension of real-valued activation functions satisfying the Lipschitz condition in real domain, complex-valued activation functions satisfying Assumption 2 are less constrained than before many papers, such as [21] and Assumption 2, in [20]. As a result of the fact, we relax Assumption 2 to moderate one that the complex-valued activation functions satisfy the Lipschitz condition as stated in Assumption 2.

Remark 2. Compared with many literatures about this research field, such as [20–22], it is not necessary for us to separate the activation functions in our paper into their real and imaginary parts or even to form an equivalent real-valued neural networks to prove the existence, uniqueness and global exponential stability of equilibrium point related to linear matrix inequality technique.

Remark 3. Not considering the discrete part of system (1), only in this way can we use a simple assumption of Lipschitz condition to prove the existence, uniqueness and global stability of equilibrium point. In our systems, the external inputs vector is always changeable with the development of time and therefore our systems are more closer to reality.

Definition 1. The equilibrium point $\bar{z}=(\bar{z_1},\dots,\bar{z_n})^T$ of system (1) is said to be globally exponentially stable, if there exist constants $\varepsilon>0$ and $M>0$ such that

$$\|z(t)-\bar{z}\|\leq M\|\phi(s)-\bar{z}\|e^{-\varepsilon(t-t_0)},$$

for all $t>0$. Where $z(t)=(z_1(t),\dots,z_n(t))^T$ is any solution of system (1) with initial value $\phi(s)=(\phi_1(s),\dots,\phi_n(s))^T,\ s\in(-\infty,0]$ and $\|\phi(s)-\bar{z}\|=\sup_{s\in[-\infty,0]}(\sum_{i=1}^n|\phi_i(s)-\bar{z}_i|^2)^{\frac{1}{2}}$.

Lemma 1. *For any vectors $x,\ y\in C^n$, positive Hermitian definite matrix $P\in C^{n\times n}$ and positive real constant $\varepsilon>0$, these following inequalities holds:*

$$x^*y+y^*x\leq\varepsilon^{-1}x^*Px+\varepsilon y^*P^{-1}y,$$

$$x^*Py+y^*Px\leq\varepsilon^{-1}x^*Px+\varepsilon y^*Py,$$

$$(x+y)^*(x+y)\leq 2x^*x+2y^*y.$$

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