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On cluster synchronization of heterogeneous systems using contraction analysis

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In this paper, we employ contraction theory to solve cluster synchronization problem under directed topology. According to the cluster structure, we develop an invariant subspace. Then we take the advantage of the complementary space to prove that the whole systems will synchronize to this invariant subspace, leading to cluster synchronization. We first deal with the linear systems which are linearly coupled under the framework of directed topology. Some sufficient conditions are given to guarantee that the coupled linear systems can achieve cluster synchronization. Moreover, the case of linearly coupled non-linear systems is also considered. Two simulation examples are given to verify our theoretical findings.

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1. Introduction

Synchronization problem has a long history [1], which has been discovered in many different fields, such as system biology, neural networks, sensor networks, formation control, and traffic management [2,3]. The studies on the complete consensus of networked multi-agent systems with the aim to reach an agreement have been widely conducted in recent years, see, e.g., [4–6]. However, in many cases the states of the coupled systems may not always synchronize to the same trajectory. A real-world complex network can be divided into some smaller subnetworks [7,8]. For example, for a flock of birds flying around in the sky, the same species of birds will achieve synchronized form, while different species of birds will not. So far, the cluster synchronization phenomenon has attracted the researchers from various disciplines of engineering and science. This phenomenon can be observed when systems synchronize to several different groups, which in the following are termed clusters. Recently, increasing attention has been paid to cluster synchronization.

There have been plenty papers working on cluster synchronization, such as [1,7–10]. In general, the network with cluster synchronization can give richer information than the network with just (complete) consensus. The cluster synchronization implies that a coupled multi-agent system is split into several clusters and all the agents in the same cluster will synchronize to the same trajectory, while the agents in different cluster may or may not synchronize to different trajectories [9]. Note that in the real network, some agents may cooperate, while some may compete. Then, the networks with both cooperate and antagonistic interactions can be denoted by signed graphs. The weights of edges in the graph can be positive/negative which are decided by the relationship between the agents, e.g., trust/distrust [11]. In most cases, the agents in the same cluster are cooperative, while competitive in different clusters. Now, a natural question is to state under what conditions associated with coupling strength and the topology of the network, the systems in each cluster will synchronize to the same trajectory. In [11], the authors consider the cluster synchronization for identical linear and non-linear system via pinning control. Some sufficient conditions are proposed to enable the systems to realize cluster synchronization under fixed and switching topology, respectively. By working in the similar framework that each cluster has a spanning tree, cluster synchronization is investigated in [12], where the authors consider nonidentical systems, Sufficient conditions are also provided to solve the cluster synchronization problem of linear and nonlinear systems, respectively. In [7,13], the authors investigate the cluster synchronization of hybrid coupled systems with communication time delay. In [14], chemically coupled and generally formulated networks cluster synchronization problem is considered. In [10], the authors drive a network to a se-

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lected cluster synchronization pattern by means of pinning control strategy. The sufficient conditions are given to guarantee the cluster synchronization.

Different from the Lyapunov method, which is commonly used in the above surveyed papers, a new method, contraction analysis, is proposed in [15–18]. In [15], contraction analysis which is derived from differential geometry and continuum mechanics, is proposed. Contraction analysis is intended for convergence but does not need to know the nominal motion *a prior*. All the trajectories corresponding to different initial conditions converge to a common one. Other related properties of contraction analysis can be found in [18]. Latter, the above works are extended to a more general case in [19], which investigates the concurrent synchronization for interacting cluster of linear and non-linear systems. Nevertheless, the authors do not consider the impact of couplings and work on an undirected network.

In view of the above inspirations, in this paper we aim to study the cluster synchronization problem with heterogeneous systems under directed topology. We focus on the cluster synchronization problem and use the contraction theory to perform the synchronization analysis. We give a geometric explanation on the emergence of cluster synchronization. In this paper, we adopt the model which has carried out a modification to the existing synchronization model.

The rest of the paper is organized as follows. In Section 2, some preliminary knowledge on graph theory and contraction theory is introduced. In Section 3, the sufficient conditions are given to guarantee the cluster synchronization of linear systems and non-linear systems, respectively. In Section 4, two illustrative examples are given to verify our results. Finally, the main results of this paper are summarized in Section 5.

Notation: Throughout this paper, let $\|x\|$ stand for the Euclidean norm of a vector x. Let $\mathrm{diag}\{\Omega_1, \ldots, \Omega_n\}$ denote the block diagonal matrix, and the jth diagonal block is square matrix Ω_j, $j = 1, \ldots, n$. A^T denote the transpose of the real matrix A. Q is a symmetric matrix; $\lambda_{\min}(Q)$ and $\lambda_{\max}(Q)$ denote the smallest and largest eigenvalues, respectively. Let I be the identity matrix of compatible dimension; $\mathbf{1}$ be the all one column of compatible dimension; $\otimes$ stand for the Kronecker product.

2. Preliminary

In this section, we revisit some classical concepts of graph theory and introduce some preliminary knowledge about contraction theory.

2.1. Graph theory

The weighted digraph with order N is denoted by $G = (\mathcal{V}, \epsilon, \mathcal{A})$, where $\mathcal{V} = \{1, \ldots, N\}$ denote the finite non-empty node set; $\epsilon \in \mathcal{V} \times \mathcal{V}$ denote the finite non-empty edge set; $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ is the weighted adjacency matrix. $a_{ij} \neq 0$ when there is a directed edge from node j to node i, otherwise $a_{ij} = 0$. Moreover, assume that $a_{ii} = 0$, $i = 1, \ldots, N$. Let $L = [l_{ij}]$ be the Laplacian matrix associated with G, with $l_{ij} = -a_{ij}$, $i \neq j$, and $l_{ii} = \sum_{k=1,k\neq i}^{N} a_{ik}$.

A directed path is a sequence of edges in a directed graph of the form $(i_1, i_2), (i_2, i_3), \ldots, (i_{q-1}, i_q)$. A directed graph is called strongly connected if for any different two agents i and j, there exists a directed path from i to j. A directed graph has a directed spanning tree if there exists at least one node, called the root node, having a directed path to all other nodes.

Without loss of generality, in what follows, we shall assume that the nodes in the graph G can be split into q $(1 \le q \le N)$ clusters $\{\mathcal{V}_1, \ldots, \mathcal{V}_q\}$ such that $\mathcal{V}_i \cap \mathcal{V}_j = \emptyset$, $\cup_{i=1}^{q} \mathcal{V}_i = \mathcal{V}$. Let G_i denote the graph of cluster $\mathcal{V}_i$. For any node $i \in \mathcal{V}$, we use $\bar{i}$ to denote which cluster the node i belongs to, i.e., $\bar{i} = \ell$ if $i \in \mathcal{V}_\ell$.

The Laplacian matrix L associated with G takes the following form:

$$\begin{bmatrix} L_{11} & \cdots & L_{1q} \\ \vdots & \ddots & \vdots \\ L_{q1} & \cdots & L_{qq} \end{bmatrix}$$

where $L_{ii} \in \mathbb{R}^{N_i \times N_i}$, $i = 1, \ldots, q$, $\sum_{i=1}^{q} N_i = N$.

Definition 1. For a directed graph G with Laplacian matrix is L, define

$$a_1(L) = \min_{x \in K} x^T L x = \min_{x \neq 0, x \perp \mathbf{1}} \frac{x^T L x}{x^T x},$$

where $K = \{x \in \mathbb{R}^n, x \perp \mathbf{1}, \|x\| = 1\}$.

In fact, a_1 can be written in another form

$$a_1(L) = \min_{x \in \mathbb{R}^{n-1}, \|Qx\|=1} x^T Q^T L Q x = \lambda_{\min}\left(\frac{1}{2} Q^T (L + L^T) Q\right),$$

where the column vectors of $Q \in \mathbb{R}^{n\times(n-1)}$ form an orthonormal basis of $\mathbf{1}^{\perp}$.

2.2. Contraction Theory

In this subsection, we will give some preliminary discussion on contraction theory. Consider a deterministic system in the form of

$$\dot{x}(t) = f(x, t), \tag{1}$$

where $f : \mathbb{R}^n \to \mathbb{R}^n$ is a nonlinear vector function and $x \in \mathbb{R}^n$ is the state vector.

The contraction theory has been vastly used in many existing literature [15–19]. In this paper, we will introduce some properties of the contraction theory for stability analysis. The contraction theory, which can be found in [15] for details, can be restated as follows.

Considering the system (1), contraction analysis is motivated by the elementary remark that talking about stability does not require to known what the nominal motion is: intuitively, a system is stable in some region if initial conditions or temporary disturbances are somehow “forgotten”. Then stability of the system can be obtained.

Firstly, we show some properties of fluid flow. the Eq. (1) can be considered as an n-dimensional fluid flow, where $\dot{x}$ is the n-dimensional “velocity” vector at the n-dimensional position x and time t. Assuming that $f(x, t)$ is smooth, then (1) yields the exact differential relation

$$\delta\dot{x} = \frac{\partial f}{\partial x}(x, t)\delta x \tag{2}$$

where δx is a virtual displacement of the two trajectories with different initial state in the flow field $\dot{x} = f(x, t)$. The squared distance between the two trajectories is defined as $\delta x^T \delta x$. Then, we will consider the differential of squared distance of the two trajectories.

$$\frac{d}{dt}(\delta x^T \delta x) = 2\delta x^T \delta\dot{x} = 2\delta x^T \frac{\partial f}{\partial x}\delta x \tag{3}$$

Definition 2. A given system (1) is contracting, if the Jacobian matrix $\frac{\partial f}{\partial x}$ satisfies

$$\frac{1}{2}\left(\frac{\partial f}{\partial x} + \frac{\partial f^T}{\partial x}\right) < -\beta I, \quad \forall x, \forall t \geq 0,$$

where β is a positive constant.

Using the transformation of coordinates, let $\delta z = \Theta(x, t)\delta x$, $M(x, t) = \Theta^T\Theta$, we can get the following results.

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