Neurocomputing 000 (2017) 1–10

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Fully distributed output regulation of high-order multi-agent systems on coopetition networks

Yanzhi Wu[a], Yiyi Zhao[b], Jiangping Hu[a,*], Bijoy Kumar Ghosh[c], Yuping Zhang[a]

[a] School of Automation Engineering, University of Electronic Science and Technology of China, Chengdu 611731, China
[b] School of Business Administration, Southwestern University of Finance and Economics, Chengdu 611130, China
[c] Department of Mathematics and Statistics, Texas Tech University, Lubbock, TX 79409-1042, USA

ARTICLE INFO

Article history:
Received 16 June 2017
Revised 17 October 2017
Accepted 1 December 2017
Available online xxx

Communicated by Jiahu Qin

Keywords:
Distributed output regulation
Bipartite consensus
Reduced-order observer
Coopetition network
Internal model

ABSTRACT

In this paper, a fully distributed output regulation problem is investigated for a linear multi-agent system with both cooperative and competitive interactions. The interaction network associated with the multi-agent system is conveniently modeled by an undirected signed graph and called coopetition network for simplicity. In most literatures on cooperative output regulation problems, it is commonly assumed that the state matrix of the exogenous system is available to all agents. In this paper, a distributed adaptive observer is developed for each agent to estimate the state and the state matrix of the exogenous system to relax the constraint. Furthermore, a reduced-order observer based controller is proposed for each agent by using an internal model approach to guarantee bipartite consensus. Finally, simulation results are provided to demonstrate the proposed control strategy.

1. Introduction

Very recently, considerable effort has been spent investigating the cooperative output regulation problem of multi-agent systems. Two kinds of control strategies, i.e., feedback control and internal model based control, have been well-known proposed to solve the problem. For example, some state feedback controllers were given in [1–3] and dynamic output feedback controllers were designed in [4–6] to solve the cooperative output regulation problem for linear multi-agent systems, respectively. As another important control strategy, some internal model based control strategies were proposed in [7–15] to solve the cooperative output regulation problem of linear multi-agent systems.

With the help of internal model approach, a distributed control strategy was proposed in [7] under the assumption that the interaction network has no cycle and [8] further extended [7] by removing the stringent assumption. With the help of internal models, a necessary condition for the synchronization between heterogeneous agents was given in [9]. A general framework was formulated for a leader-follower consensus problem with the help of distributed internal model principle in [10]. In [11], a high-gain approach was proposed to deal with the output synchronization problem for heterogeneous multi-agent systems with the assumption that the agents are non-introspective. Output regulation problem for heterogeneous networks of introspective agents was also investigated in [12]. Output regulation for uncertain heterogeneous systems was further studied in [13], where the uncertainties were arbitrarily but bounded. A general framework was formulated for a leader-follower consensus problem with the help of distributed internal model principle in [10]. A fully distributed controller was designed in [14] to overcome the restriction on global information. The output regulation problem was solved by combining a feedback passivity method and the internal model principle in [15].

Although the cooperative output regulation problem of high order multi-agent systems has been investigated extensively, to the best of the authors' knowledge, few results consider output regulation problem for multi-agent systems with antagonistic interactions. In fact, cooperation and competition coexist widely in natural or man-made complex systems. Many examples with antagonistic interactions exist, such as opinion dynamics, the strategic management, game theory, competitive activities, and so on. A static signed graph was introduced to model the coopetition network and a distributed control strategy was proposed to guarantee bipartite consensus in [16]. [17] extended to study Altafini's model on opinion dynamics over general directed time-varying graphs. Both static network topologies and dynamically changing topologies were considered in [18]. [19] introduced the

* Corresponding author.
E-mail address: hjp_lzu@163.com (J. Hu).

https://doi.org/10.1016/j.neucom.2017.12.001
0925-2312/

concept of interval bipartite consensus and proved the interval bipartite consensus was achieved if the associated signed digraph had a spanning tree. Bipartite consensus means that all agents reach a unique value with identical magnitude but opposite sign. Furthermore, three kinds of group patterns, i.e., consensus, bipartite consensus and fragmentation, were investigated for coopetition networks in [20]. Furthermore, [21] proved several convergence results for several models with antagonisms. Some extensions were made in [22] for a consensus problem of discrete-time multi-agent systems with antagonistic interactions and switching topologies. Particularly, some recent papers proposed feedback controller or adaptive controller for high-order multi-agent systems with antagonistic interactions in [23–25], respectively. By using a low gain feedback technique, leaderless bipartite consensus with input saturation was considered for directed graph in [27]. H_∞ group consensus was considered in [28] for first-order multi-agent systems with uncertainty ad disturbances. Group synchronization for linearly or nonlinearly coupled multi-agent systems was also investigated in [29]. Different from bipartite consensus where all agents reach a unique value with identical magnitude but opposite sign, scaled consensus means all agents reach asymptotically suitable proportions respect to some prescribed scales. Both finite time and delayed scaled consensus problems were studied in [30] and [31], respectively. Moreover, fixed-time group consensus were also been investigated in [32] and [33], where agents in different subgroups were allowed to maintain antagonistic relationships.

Motivated by the observations above, this paper handles a coopetitive output regulation problem for linear multi-agent systems by assuming that only a part of agents can get the information (for example, the state, the output and the state matrix) of the exogenous system. The contributions of the paper are threefold. First, an output regulation problem is formulated for coopetitive multi-agent systems. Traditional definitions for cooperative output regulation are not valid any longer. Second, a distributed adaptive observer is developed for each agent to estimate the state of the exogenous system. At the same time, an adaptive gain scheduling strategy is proposed to relax the constraint that the state matrix of the exogenous system is available to all agents. Third, since not all agents can use the output of the exogenous system, a reduced-order observer-based controller is designed for each agent to guarantee bipartite consensus by using the concept of *p*-copy internal model.

The remainder of this paper is organized as follows. In Section 2, some useful preliminary results are reviewed and the mathematical description of coopetitive output regulation problem is formulated. In Section 3, distributed adaptive observers and bipartite consensus controllers are respectively proposed for each agent. The convergence of the output regulation error of the closed-loop multi-agent system is analyzed. A simulation example is provided to validate the effectiveness of the proposed control strategy in Section 4. Finally, Section 5 summarizes the results of the paper.

Throughout this paper, the following notations are used. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ represent the set of n-dimensional Euclidean space and $n\times m$ real matrices, respectively. $\otimes$ represents the Kronecker product. A matrix $P>0$ $(P\geq 0)$ means that P is positive definite (positive semidefinite). The identity matrix with compatible dimension is written as I. For an arbitrary matrix $A\in\mathbb{R}^{n\times n}$, $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ respectively denote the maximum and the minimum eigenvalues of the matrix A. diag$(\cdot)$ and block-diag$(\cdot)$ denote a diagonal matrix and a block diagonal matrix, respectively. col$(\cdot)$ denotes a concatenation. sign$(\cdot)$ denotes a sign function.

2. Mathematical preliminaries

2.1. Some preliminaries

The coopetition network associated with a multi-agent system can be conveniently described by an undirected signed graph $\mathcal{G}=\{\mathcal{V},\mathcal{E},\mathcal{A}\}$, where $\mathcal{V}=\{v_1,v_2,\ldots,v_N\}$ represents the node set, $\mathcal{E}\subset\mathcal{V}\times\mathcal{V}$ represents the edge set, and $\mathcal{A}=(\mathcal{A}_{ij})_{N\times N}$ is the adjacency matrix. The element $\mathcal{A}_{ij}$ is nonzero if there is an edge between node i and node j. Specifically, if $\mathcal{A}_{ij}>0$, the relationship between agent i and agent j is cooperative, otherwise, the relationship is competitive when $\mathcal{A}_{ij}<0$. The signed graph $\mathcal{G}$ is supposed to be simple, i. e., $\mathcal{A}_{ii}=0$ for $i=1,2,\ldots,N$. The Laplacian matrix $\mathcal{L}$ is defined by

$$\mathcal{L}=\mathcal{C}-\mathcal{A}, \tag{1}$$

where $\mathcal{C}=\mathrm{diag}(\sum_{j=1}^{N}|\mathcal{A}_{1j}|,\ldots,\sum_{j=1}^{N}|\mathcal{A}_{Nj}|)$.

A path with length $l-1$ is an alternating sequence $(i_1,i_2),(i_2,i_3),\ldots,(i_{l-1},i_l)$ with distinct nodes. If $i_1=i_l$, the path can form a cycle. If there exists a path between every pair of nodes, then the graph $\mathcal{G}$ is connected. Furthermore, if there exists a path from a node to all the other nodes, then the graph $\mathcal{G}$ is said to have a spanning tree. In a signed graph, both positive and negative edges exist in a cycle. The cycle is said to be positive if the product of the edge weights in the cycle is positive. If all the cycles in the signed graph $\mathcal{G}$ are positive, $\mathcal{G}$ is strongly structurally unbalanced. If one of its cycles is negative, $\mathcal{G}$ is structurally unbalanced. $\mathcal{G}$ is vacuously balanced if no cycles in the signed graph $\mathcal{G}$. A signed graph $\mathcal{G}$ is structurally balanced if it is strongly structurally balanced or vacuously balanced. If the signed graph $\mathcal{G}$ can be divided into two subgraphs with node sets $\mathcal{V}_1$ and $\mathcal{V}_2$ which satisfy $\mathcal{V}_1\cup\mathcal{V}_2=\mathcal{V},\mathcal{V}_1\cap\mathcal{V}_2=\emptyset$ and $\mathcal{A}_{ij}\geq 0,\forall i,j\in\mathcal{V}_q$ $(q\in\{1,2\})$, $\mathcal{A}_{ij}\leq 0,$ $\forall i\in\mathcal{V}_q, j\in\mathcal{V}_r, q\neq r(q,r\in\{1,2\})$, then the graph $\mathcal{G}$ is structurally balanced. The adjacency matrix $\mathcal{A}$ of a structurally balanced graph can be rewritten in the following form by reordering of the agents

$$\mathcal{A}=\begin{pmatrix}\bar{\mathcal{A}}_{11} & \bar{\mathcal{A}}_{12}\\ \bar{\mathcal{A}}_{21} & \bar{\mathcal{A}}_{22}\end{pmatrix}, \tag{2}$$

where $\bar{\mathcal{A}}_{11}$ and $\bar{\mathcal{A}}_{22}$ are nonnegative submatrices, while $\bar{\mathcal{A}}_{12}=\bar{\mathcal{A}}_{21}$ is a nonpositive submatrix. A gauge transformation is defined by $S=\mathrm{diag}(s_1,s_2,\ldots,s_N)\in\mathbb{R}^{N\times N}$, where the diagonal element s_i equals 1 or -1 .

Lemma 1. *[16] For the signed Laplacian matrix $\mathcal{L}$, the following facts are equivalent:*

(1) *$\mathcal{G}$ is structurally balanced;*
(2) *$\mathcal{A}$ is equivalent to a nonnegative matrix by using a gauge transformation;*
(3) $\lambda_{\min}(\mathcal{L})=0$.

Remark 1. A signed graph is a graph in which each edge has a positive or negative sign and can be regarded as an extension of unsigned graphs. The signed Laplacian matrix is associated with a signed graph and thus is also an extension of a Laplacian matrix relevant to an unsigned graph. The two Laplacian matrices can be similar when the associated signed graph is structurally balanced. Additionally, it is noted that the signed graph is assumed to be undirected, thus the signed Laplacian matrix $\mathcal{L}$ is symmetric.

When there exists a leader for a multi-agent system, the leader is denoted by node 0 and thus a new graph $\bar{\mathcal{G}}=\mathcal{G}\cup\{0\}$. If agent i can get information from the leader, the weight