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New results for exponential stability of complex-valued memristive neural networks with variable delays

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ABSTRACT

This paper studies the dynamic behaviors of complex-valued memristive neural networks with variable delays. By using *M*-matrix, redinequality technique and Lyapunov function, sufficient conditions are proposed to guarantee the existence, uniqueness and global exponential stability for the equilibrium point of complex-valued memristive neural networks with variable delays. The proposed results are easy to be checked and are helpful in qualitative analysis for some complex-valued nonlinear delayed systems. Two numerical examples are given to demonstrate the effectiveness of theoretical results.

1. Introduction

The dynamic behaviors of neural networks have been widely investigated in the last decades due to their potential applications in signal processing, digital selection, pattern recognition, associative memory and so on [1–8]. However, with the expansion of the requirement in practice, some problems can only be solved successfully in the complex number field. Thus, complex-valued neural networks (CVNNs) have been concerned by many researchers in recent years. As the extension of real-valued neural networks, CVNNs have complex-valued states, complex-valued connection weights and complex-valued activation functions. Therefore, the real-valued neural networks are the special case of CVNNs. Especially, the activation functions of real-valued neural networks are chosen to be smooth and bounded. According to the Liouville theorem, every smooth and bounded activation function reduces a constant in complex number field [9,10], hence, the selection of appropriate activation function is the primary challenge in CVNNs. Many important and classical results in CVNNs have been proposed in recent years. For instance, Xu et al. [11] presented sufficient conditions to guarantee exponential stability of CVNNs with mixed delays. Several sufficient conditions for multiple μ-stability of CVNNs with unbounded time delays have been obtained by Rakkiyappa et al. [12]. Stability analysis of CVNNs with probabilistic time delays have been processed by Song et al. [13]. Zhou and Song [14] discussed the boundedness and complete stability of CVNNs with time delay. Gong et al. [15] investigated the global asymptotic stability for the CVNNs.

Memristor is a new circuit element which is firstly proposed in 1971 [16] , and Hewlett–Packard Laboratory scientists announced the development of a practical memristor in 2008 [17,18]. Since then, researchers have replaced resistor with memristor in very large scale integration circuits to build memristive neural networks, and investigate the dynamic behaviors of memristive neural networks in [19–24]. Similarly, we can also introduce the memristive weights into CVNNs to construct complex-valued memristive neural networks (CVMNNs).

As is well known, time delay exists inevitably in neurodynamic systems for the finite switching speed of amplifiers or information processing. And time delay may lead to bad performances, including oscillation, and even instability in CVMNNs. Hence, the dynamic analysis for a class of CVMNNs with time delay is a hot researched topic in recent years [25–28]. Li et al. [25] presented several sufficient conditions for global dissipativity and global exponential dissipativity of CVMNNs. Passivity analysis of CVMNNs is investigated in [26,27]. Rakkiyappan et al. [28] proposed some criteria to assure the finite-time stability of fractional-order CVMNNs. In most cases, delay is variable, the absolutely fixed delay is very rare, and it is just an idealized approximation of the variable delay. So, the obtained sufficient conditions may be conservative. Therefore, it is very important and necessary to develop the

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available probability distribution and obtain a larger allowable variation range of the delay. However, few literatures involved with the analysis of global exponential stability of CVNNs [10,29]. Several sufficient conditions have been reported to assure the exponential stability of CVNNs with constant delay in [10]; without time delays [29]. As far as we know, the global exponential stability of CVMNNs with variable delays has not been studied yet in the present literatures. Researching the problem of global exponential stability of CVMNNs with variable delays is a complicated task, when comparing to study same problem for CVMNNs with constant delays. This situation stimulates our present study. Motivated by the above discussion, the main goal of this paper is to propose new sufficient condition for global exponential stability of CVMNNs with variable delays.

The model of CVMNNs with variable delays can be described as follows:

$$\dot{z}_k(t) = -d_k z_k(t) + \sum_{l=1}^{n} a_{kl}\big(z_l(t)\big) f_l\big(z_l(t)\big) + \sum_{l=1}^{n} b_{kl}\big(z_l(t)\big) g_l\big(z_l(t-\tau_l(t))\big) + u_k(t), \tag{1}$$

for $t \geq 0$, $k = 1, 2, \ldots, n$, where $z_k(t)$ is the complex-valued state vector of the kth neuron and $z(t) = (z_1(t), z_2(t), \ldots, z_n(t))^T \in \mathcal{C}^n$. $d_k \geq 0$ is the self-feedback connection weight, $a_{kl}(z_l(t))$ and $b_{kl}(z_l(t))$ are complex-valued connection weights, $f(z(\cdot)) = (f_1(z_1(\cdot)), f_2(z_2(\cdot)), \ldots, f_n(z_n(\cdot)))^T$ and $g(z(\cdot)) = (g_1(z_1(\cdot)), g_2(z_2(\cdot)), \ldots, g_n(z_n(\cdot)))^T$ represent the complex-valued activation functions with $f(0) = g(0) = 0$. $u_k(t)$ denotes the external input vector. $\tau_l(t)$ stands for the variable time delay and it is supposed to be differential and satisfies $0 \leq \tau_l(t) \leq \tau$, $\dot{\tau}_l(t) \leq \mu < 1$, where τ and μ are positive constants. We defined

$$d_k = \frac{1}{\mathfrak{C}_k}\left[\sum_{l=1}^{n} \text{sign}_{kl}(\mathbb{M}_{kl} + \mathbb{N}_{kl}) + \frac{1}{\mathfrak{R}_k}\right],$$

$$a_{kl}\big(z_l(t)\big) = \text{sign}_{kl}\frac{\mathbb{M}_{kl}}{\mathfrak{C}_k}, \quad b_{kl}\big(z_l(t)\big) = \text{sign}_{kl}\frac{\mathbb{N}_{kl}}{\mathfrak{C}_k},$$

$$\text{sign}_{kl} = \begin{cases} 1, & k \neq l, \\ -1, & k = l, \end{cases}$$

where $\mathbb{M}_{kl}$ and $\mathbb{N}_{kl}$ represent the memductances of memristor $\mathcal{G}_{kl}$ and $\mathcal{F}_{kl}$, $\mathcal{G}_{kl}$ stands for the memristor between the activation function $f_l(z_l(t))$ and $z_l(t)$, $\mathcal{F}_{kl}$ represents the memristor between the activation function $g_l\big(z_l(t-\tau_l)\big)$ and $z_l(t)$, $\mathfrak{C}_k$ is a capacitor, $\mathfrak{R}_k$ is the resistor parallel to the capacitor $\mathfrak{C}_k$.

According to the features of the memristor, we get

$$a_{kl}\big(z_l(t)\big) = \begin{cases} \acute{a}_{kl}, & \text{sign}_{kl}\frac{df_l(z_l(t))}{dt} - \frac{dz_k(t)}{dt} \leq 0, \\ \grave{a}_{kl}, & \text{sign}_{kl}\frac{df_l(z_l(t))}{dt} - \frac{dz_k(t)}{dt} > 0, \end{cases}$$

$$b_{kl}\big(z_l(t)\big) = \begin{cases} \acute{b}_{kl}, & \text{sign}_{kl}\frac{dg_l(z_l(t-\tau_l(t)))}{dt} - \frac{dz_k(t)}{dt} \leq 0, \\ \grave{b}_{kl}, & \text{sign}_{kl}\frac{dg_l(z_l(t-\tau_l(t)))}{dt} - \frac{dz_k(t)}{dt} > 0, \end{cases}$$

for $k, l = 1, 2, \ldots, n$, where the switching jumps $\acute{a}_{kl}, \grave{a}_{kl}, \acute{b}_{kl}$ and $\grave{b}_{kl}$ are constants.

Remark 1. Different from state-independent switched nonlinear systems [30–35], the memristive systems are a class of state-dependent switched nonlinear systems because the memristive weights are not fixed. Therefore, when $a_{kl}(z_l(t))$ and $b_{kl}(z_l(t))$ in model (1) become constants, complex-valued memristive neural networks will reduce to a class of conventional complex-valued neural networks [10–15].

Remark 2. The memristive connection weights $a_{kl}(z_l(t))$, $b_{kl}(z_l(t))$ are discontinuous. The solution of system (1) cannot be found in classical manner. Filippov proposed a new method for analysis differential equations with discontinuous right-hand sides [36]. According to [36] , the solution of it has the same solution set as a certain differential inclusion.

2. Preliminaries

In this paper, the solutions of all the systems considered below are intend in Filippov's sense [36]. $\mathcal{R}^n$ and $\mathcal{C}^n$ denote n-dimensional Euclidean space and complex space, respectively. co$\{\digamma_1, \digamma_2\}$ represents closure of the convex hull of $\mathcal{C}^n$ generated by complex numbers $\digamma_1$ and $\digamma_2$. For complex-valued function $z = x + iy \in \mathcal{C}$, where i is the imaginary unit and satisfy $i = \sqrt{-1}$, $x, \ y \in \mathcal{R}$. Let $|z| = (|z_1|, |z_2|, \ldots, |z_n|)^T$, $\|\cdot\|$ denotes the Euclidean vector norm.

According to [19] , by using the theories of set valued map and differential inclusions, from (1), we have

$$\dot{z}_k(t) \in -d_k z_k(t) + \sum_{l=1}^{n} \text{co}[a_{kl}^+, a_{kl}^-] f_l\big(z_l(t)\big) + \sum_{l=1}^{n} \text{co}[b_{kl}^+, b_{kl}^-] g_l\big(z_l(t-\tau_l(t))\big) + u_k(t), \tag{2}$$

$k = 1, 2, \ldots, n$, where $a_{kl}^+ = \max\{\acute{a}_{kl}, \grave{a}_{kl}\}$, $a_{kl}^- = \min\{\acute{a}_{kl}, \grave{a}_{kl}\}$, $b_{kl}^+ = \max\{\acute{b}_{kl}, \grave{b}_{kl}\}$, $b_{kl}^- = \min\{\acute{b}_{kl}, \grave{b}_{kl}\}$. Or equivalently, there exist measurable functions $\widehat{a}_{kl} \in \text{co}[a_{kl}^+, a_{kl}^-]$, $\widehat{b}_{kl} \in \text{co}[b_{kl}^+, b_{kl}^-]$, such that

$$\dot{z}_k(t) = -d_k z_k(t) + \sum_{l=1}^{n} \widehat{a}_{kl} f_l\big(z_l(t)\big) + \sum_{l=1}^{n} \widehat{b}_{kl} g_l\big(z_l(t-\tau_l(t))\big) + u_k(t), \quad t \geq 0. \tag{3}$$

The above system can be expressed by matrix form as follow:

$$\dot{z}(t) = -Dz(t) + Af\big(z(t)\big) + Bg\big(z(t-\tau(t))\big) + U, \tag{4}$$

where $D = \text{diag}(d_1, \ldots, d_n) \in \mathcal{R}^{n\times n}$, $A = (\widehat{a}_{kl})_{n\times n} \in \mathcal{C}^{n\times n}$, $B = (\widehat{b}_{kl})_{n\times n} \in \mathcal{C}^{n\times n}$, $U = (u_1(t), u_2(t), \ldots, u_n(t))^T \in \mathcal{C}^n$.

Let $z_l = x_l + iy_l$, then the activation function $f_l(z_l)$ and $g_l(z_l)$ can be separated into its real and imaginary parts as

$$f_l(z_l) = f_l^R(x_l, y_l) + if_l^I(x_l, y_l),$$
$$g_l(z_l) = g_l^R(x_l, y_l) + ig_l^I(x_l, y_l), \tag{5}$$

where $f_l^R(x_l, y_l)$, $f_l^I(x_l, y_l)$, $g_l^R(x_l, y_l)$, $g_l^I(x_l, y_l) : \mathcal{R}^2 \to \mathcal{R}$, and $f_l^R(0,0) = f_l^I = (0,0) = g_l^R(0,0) = g_l^I(0,0) = 0$ is hold. For activation functions $f_l^R(x_l, y_l)$, $f_l^I(x_l, y_l)$, $g_l^R(x_l, y_l)$, and $g_l^I(x_l, y_l)$, we need the following assumption.

Assumption 1. We consider $f_l(\cdot, \cdot)$ and $g_l(\cdot, \cdot)$ with the form of (5) satisfying the following conditions.

I. The partial derivatives of $f_l(\cdot, \cdot)$ with respect to $x, y : \partial f_l^R/\partial x$, $\partial f_l^R/\partial y$, $\partial f_l^I/\partial x$ and $\partial f_l^I/\partial y$ are exist and continuous.

II. The partial derivatives $\partial f_l^R/\partial x, \partial f_l^R/\partial y, \partial f_l^I/\partial x$ and $\partial f_l^I/\partial y$ are bounded, that is to say, there exist positive constants $h_l^{RR}, h_l^{RI}, h_l^{IR}$ and h_l^{II} such that

$$|\partial f_l^R/\partial x| \leq h_l^{RR}, \quad |\partial f_l^R/\partial y| \leq h_l^{RI},$$
$$|\partial f_l^I/\partial x| \leq h_l^{IR}, \quad |\partial f_l^I/\partial y| \leq h_l^{II}.$$

III. Similarly, we can obtain

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