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Brief papers

# Delay-dependent state estimation for neural networks with time-varying delay

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ABSTRACT

This paper focuses on delay-dependent state estimation problem for neural networks with a time-varying delay. An improved delay-dependent criterion is established to estimate the neuron states through available output measurements such that the dynamics of the estimation error is globally asymptotically stable. The derivative of Lyapunov–Krasovskii functional is bounded by introducing a free-matrix-based integral inequality. A modified cone complementarity linearization (CCL) algorithm is presented to compute the state estimator parameter in obtained matrix inequalities, rather than linear matrix inequalities (LMIs). Finally, two numerical examples are given to demonstrate the effectiveness and the merits over the existing ones.



## 1. Introduction

In the past decade, neural networks have been found successful applications in various areas such as signal processing, pattern recognition, and combinatorial optimization. As time-delays are frequently encountered in many systems and they are often a source of instability and oscillations in neural networks, dynamic characters of neural networks may be changed. Therefore, neural networks with time-delays have received more attentions, and both delay-independent and delay-dependent criteria have been established to verify their asymptotical or exponential stability [1–6].

On the other hand, neuron states are not often completely available in the network outputs in many applications. As a result, in order to utilize the neuron states of the neural networks, [7] firstly studied the state estimation problem for neural networks with a time-varying delay through available output measurements. Since then, the neuron state estimation problem has attracted much attentions [8–14].

For neural networks with time-delays, [8] studied the state estimation problem and derived some delay-dependent state estimation conditions by using a free-weighting matrix (FWM) method which introduces some free matrices, and increases the computation complexity. To overcome this problem, some integral inequality methods were proposed and many results were derived based on these inequalities. For example, the robust state estimation problem for a class of uncertain delayed neural networks was considered with a bounding technique [9]. Then, in [10], a criterion was formulated by Jensen inequality [15] to guarantee the existence of a desired state estimator. To reduce the conservatism of the delay-dependent conditions, [14] constructed an augmented LKF [16,17] with a secondary-partition approach and then, they used a less conservative Wirtinger inequality [18] compared with the Jensen inequality to realize some conservatism-reduced state estimation conditions for delayed neural networks. Recently, a free-matrix-based integral inequality was proposed in [19–23], which contains the Wirtinger inequality. Moreover, a relaxed integral inequality [24] was proved to be less conservative compared with the Wirtinger inequality combined with reciprocally convex approach [25].

It should be noted that these new inequalities have not been used to solve the problem of state estimation for neural networks with a time-varying delay. In addition, the obtained criteria for state estimation in existing literature were mostly presented in terms of matrix inequalities, rather than LMIs, which corresponds to a nonlinear programming problem. In [8], this problem is dealt with a CCL algorithm [26]. However, the stopping conditions

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of initial CCL algorithm for iterating are too strict to decrease iterations for determining the value of state estimator parameters. It is very time consuming.

In this paper, by introducing the integral inequality called free-matrix-based integral inequality, an improved delay-dependent state estimation criterion for neural networks with a time-varying delay is established. As a result, a modified CCL algorithm with a new stopping condition for iterating is given to obtain the state estimator. Finally, two numerical examples are used to demonstrate the effectiveness and merits of the present method.

*Notations*: Throughout this paper, the superscripts $T$ and $-1$ mean the transpose and the inverse of a matrix, respectively; $\mathbb{R}^n$ denotes the $n$ dimensional Euclidean space; $\mathbb{R}^{n\times m}$ is the set of all $n\times m$ real matrices; $P>0$ $(\geq 0)$ means that $P$ is a real symmetric and positive definite (semi-positive definite) matrix; diag{...} denotes a block diagonal matrix; symmetric term in a symmetric matrix is denoted by $\star$ and $\mathrm{Sym}\{X\}=X+X^T$.

## 2. Problem formulation

Consider the following neural network with a time-varying delay

$$\dot{x}(t)=-Ax(t)+W_0g(x(t))+W_1g(x(t-d(t)))+U, \quad (1)$$

where $x(\cdot)=[x_1(\cdot),x_2(\cdot),\ldots,x_n(\cdot)]^T\in R^n$ is the neuron state vector, $g(x(\cdot))=[g_1(x_1(\cdot)),g_2(x_2(\cdot)),\ \ldots,g_n(x_n(\cdot))]^T$ denotes the neuron activation function with $g(0)=0$ and $U=[U_1,U_2,\ldots,U_n]$ is a constant vector. $A=\mathrm{diag}\{a_1,\ a_2,\ \ldots,\ a_n\}$ is a diagonal matrix with entries $a_i>0$. $W_0$ and $W_1$ are the connection weight matrix and the delayed connection weight matrix, respectively. The delay, $d(t)$, is a time-varying differentiable function satisfying

$$0\leq d(t)\leq h \quad (2)$$

and

$$\dot{d}(t)\leq\mu, \quad (3)$$

where $h$ and $\mu$ are scalar constants. Similar to those in [7], it is assumed that the neuron activation function, $g(\cdot)$, satisfies the following Lipschitz condition

$$|g(x)-g(y)|\leq|G(x-y)|, \quad (4)$$

where $G\in R^{n\times n}$ is a known constant matrix.

In the following, we shall try to find an efficient estimation algorithm to observe the neuron states from the available network outputs. Therefore, the network measurements are assumed to satisfy

$$y(t)=Cx(t)+f(t,x(t)), \quad (5)$$

where $y(t)\in R^m$ is the measurement output, $C$ is a known constant matrix with appropriate dimension. $f$ is the neuron-dependent nonlinear disturbances on the network outputs, and satisfies the following Lipschitz condition

$$|f(t,x)-f(t,y)|\leq|F(x-y)|, \quad (6)$$

where the constant matrix $F\in R^{n\times n}$ is also known.

The full-order state estimator is of the form

$$\begin{aligned}\dot{\hat{x}}(t)&=-A\hat{x}(t)+W_0g(\hat{x}(t))+W_1g(\hat{x}(t-d(t)))\\&\quad+U+K\big[y(t)-C\hat{x}(t)-f(t,\hat{x}(t))\big],\end{aligned} \quad (7)$$

where $\hat{x}(t)$ is the estimation of the neuron state, and $K\in R^{n\times m}$ is the estimator gain matrix to be designed.

Let the error state be

$$e(t)=x(t)-\hat{x}(t) \quad (8)$$

then it follows from (1), (5) and (7) that

$$\dot{e}(t)=(-A-KC)e(t)+W_0\psi(t)+W_1\psi(t-d(t))-K\phi(t), \quad (9)$$

where

$$\psi(t)=g(x(t))-g(\hat{x}(t))$$
$$\phi(t)=f(t,x(t))-f(t,\hat{x}(t)).$$

It is clear from (4) and (6) that

$$\psi^T(t)\psi(t)=|g(x(t))-g(\hat{x}(t))|^2\leq|Ge(t)|^2=e^T(t)G^TGe(t) \quad (10)$$

$$\phi^T(t)\phi(t)=|f(t,x(t))-f(t,\hat{x}(t))|^2\leq e^T(t)F^TFe(t). \quad (11)$$

The following lemmas will be employed to derive the new result.

**Lemma 1** (Free-matrix-based integral inequality [19,21]). *Let $x$ be a differentiable function $[\alpha,\beta]\to\mathcal{R}^n$. For symmetric matrices $\bar{R}\in\mathcal{R}^{n\times n}$, and $\bar{Z}_1,\bar{Z}_3\in\mathcal{R}^{3n\times3n}$, and any matrices $\bar{Z}_2\in\mathcal{R}^{3n\times3n}$, and $\bar{N}_1$, $\bar{N}_2\in\mathcal{R}^{3n\times n}$ satisfying*

$$\bar{\Phi}=\begin{bmatrix}\bar{Z}_1&\bar{Z}_2&\bar{N}_1\\\star&\bar{Z}_3&\bar{N}_2\\\star&\star&\bar{R}\end{bmatrix}\geq0$$

*the following inequality holds*

$$-\int_\alpha^\beta\dot{x}^T(s)\bar{R}\dot{x}(s)ds\leq\varpi^T\Omega\varpi, \quad (12)$$

*where*

$$\varpi=\begin{bmatrix}x^T(\beta)&x^T(\alpha)&\frac{1}{\beta-\alpha}\int_\alpha^\beta x^T(s)ds\end{bmatrix}^T$$
$$\begin{aligned}\Omega&=(\beta-\alpha)\Big(\bar{Z}_1+\frac{1}{3}\bar{Z}_3\Big)\\&\quad+\mathrm{Sym}\{\bar{N}_1(\bar{e}_1-\bar{e}_2)+\bar{N}_2(2\bar{e}_3-\bar{e}_1-\bar{e}_2)\}\end{aligned}$$
$$\bar{e}_1=[I\ 0\ 0],\quad\bar{e}_2=[0\ I\ 0],\quad\bar{e}_3=[0\ 0\ I]$$

## 3. Main results

First, suppose that the estimator gain matrix $K$ is given. The following Theorem is obtained for the globally asymptotical stability of system (9).

**Theorem 1.** *Given scalars $h>0$ and $\mu$, system (9) with time-varying delay $d(t)$ satisfying (2) and (3) is globally asymptotically stable for given estimator gain matrix $K$ if there exist positive-definite symmetric matrices $P\in\mathcal{R}^{n\times n}$, $Q_i\in\mathcal{R}^{n\times n}$, $i=1,2,3$, $R\in\mathcal{R}^{n\times n}$, symmetric matrices $X_1,X_3,Y_1,Y_3\in\mathcal{R}^{3n\times3n}$, and any matrices $X_2,Y_2\in\mathcal{R}^{3n\times3n}$, and $N_1,N_2,M_1,M_2\in\mathcal{R}^{3n\times n}$, and scalars $\varepsilon_j>0$, $j=1,2,3$, such that the following LMIs hold*

$$\Xi_1=\begin{bmatrix}\Phi+h\Phi_1&h\Theta^TR\\\star&-hR\end{bmatrix}<0 \quad (13)$$

$$\Xi_2=\begin{bmatrix}\Phi+h\Phi_2&h\Theta^TR\\\star&-hR\end{bmatrix}<0 \quad (14)$$

$$\Psi_1=\begin{bmatrix}X_1&X_2&N_1\\\star&X_3&N_2\\\star&\star&R\end{bmatrix}\geq0 \quad (15)$$

$$\Psi_2=\begin{bmatrix}Y_1&Y_2&M_1\\\star&Y_3&M_2\\\star&\star&R\end{bmatrix}\geq0 \quad (16)$$