ARTICLE IN PRESS
JID: NEUCOM [m5G;October 9, 2017;10:13]

Neurocomputing 000 (2017) 1–11

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Analysis and pinning control for passivity of multi-weighted complex dynamical networks with fixed and switching topologies

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ARTICLE INFO

Article history:
Received 3 April 2017
Revised 17 July 2017
Accepted 13 September 2017
Available online xxx

Communicated by Haijun Jiang

Keywords:
Multi-weights
Complex dynamical networks
Passivity
Pinning control

ABSTRACT

In this paper, we respectively discuss passivity and pinning passivity of multi-weighted complex dynamical networks. By employing Lyapunov functional approach, several passivity criteria for the complex dynamical network with fixed topology and multi-weights are established. In addition, under the designed pinning adaptive state feedback controller, some sufficient conditions are obtained to ensure the passivity of the multi-weighted complex dynamical network with fixed topology. Furthermore, similar methods are used to derive several criteria for passivity and pinning passivity of complex dynamical networks with switching topology and multi-weights. Finally, two numerical examples with simulation results are given to show the correctness of the obtained passivity criteria.

1. Introduction

In 1968, passivity was firstly raised by Bevelevich in the circuit analysis [1] which can ensure the internal stability of the system. Since then passivity has received extensive attention and it has been widely used in many fields such as feedback control [2,3], magnetic suspension system [4], nonlinear descriptor system [5], induction motors [6,7], and synchronization [8]. Therefore, investigation on the passivity and control problems of different systems is very meaningful.

Over the past two decades, complex networks have aroused the interest of many researchers in physics, economics, biology, mathematics, engineering, and so on. More recently, many authors have studied the dynamical behaviors of complex networks. Especially, as one of the most important dynamical behaviors in complex networks, passivity has been widely investigated by researchers. To our knowledge, the main reason for this is that passivity is a very effective tool to analyze the stability and synchronization of complex dynamical networks [9–14]. Fang and Zhao [9] considered the input passivity of complex delayed dynamical networks with output coupling by using Lyapunov functional method. In [10], the authors studied passivity and synchronization of complex dynamical networks including the possible presence of communication time delays. Ren et al. [11] introduced a complex delayed dynamical network with spatial diffusion coupling, and respectively considered passivity and pinning passivity of the proposed network model.

However, in the above mentioned results about passivity of complex dynamical networks, the authors always assume that the dimensions of input and output are the same. But in many real systems, the dimensions of input and output are different. Unfortunately, very few researchers have taken this problem into consideration [15,16]. In [15], the authors analyzed passivity for two coupled reaction-diffusion neural networks with different dimensions of input and output. Wang et al. [16], respectively, investigated the passivity of directed and undirected coupled neural networks with reaction-diffusion terms by using the designed adaptive laws. Furthermore, in practical applications, the connection between network nodes often changes by switches due to external disturbance and limited communications [17–20]. Thus, it is also interesting to consider the passivity of complex dynamical networks with switching topology. To our knowledge, the passivity of complex dynamical networks with switching topology and different dimensions of input and output vectors has not yet been studied.

As everyone knows, we can get in touch with others through many channels such as Facebook, Wechat, letters and so on, and each contact method stands for different coupling. In this case, the social network can be modeled by complex dynami-

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https://doi.org/10.1016/j.neucom.2017.09.037
0925-2312/

Please cite this article as: X.-X. Zhang et al., Analysis and pinning control for passivity of multi-weighted complex dynamical networks with fixed and switching topologies, Neurocomputing (2017), https://doi.org/10.1016/j.neucom.2017.09.037

cal network model with multi-weights. Obviously, multi-weighted complex networks can better reflect the relationship among persons and more properly describe the real social network. Practically, many real-world networks should be modeled by multi-weighed network models, such as communication networks, complex biology networks, transportation networks. But, minority of researchers have investigated dynamical behaviors of complex networks with multi-weights [21–23]. In [21], the authors considered the global synchronization of the public traffic roads networks with multi-weights based on the Lyapunov stability theory. The synchronization problem of uncertain complex networks with multiple coupled time-varying delays is concerned by Zhao et al. [22] based on robust adaptive principle. However, very few results about the passivity of complex dynamical networks with multi-weights have been reported.

In many situations, complex dynamical networks are not passive, thus some control strategies are required to make sure the passivity of networks. Nevertheless, it is very tough to control all nodes in the networks, especially in a large-scale network. For this phenomenon, many pinning control schemes have been presented to ensure that complex networks achieve the desired dynamical behaviors [24–34]. Li and Yang [24] verified the global pinning synchronization problem for the given complex dynamical network model by planning the adaptive controller and using pinning control scheme. In [25], adaptive pinning synchronization problem on stochastic complex dynamical networks is researched by Li and Yang under the help of algebraic graph theory. Jin and Yang [26] utilized adaptive control techniques and solved the problem of pinning control of synchronization for nonlinearly coupled complex networks. To our knowledge, few researchers have focused attention on the pinning passivity problem [11,12]. In [12], Ren et al. considered the main problems of passivity and pinning passivity for coupled delayed reaction-diffusion neural networks by constructing suitable pinning controllers. Unfortunately, the problem of pinning passivity for multi-weighted complex dynamical networks has not been considered especially for multi-weighted complex dynamical networks with switching topology.

To the best of our knowledge, this is the first paper to consider the passivity and pinning passivity of multi-weighted complex dynamical networks with fixed and switching topologies, which are very significant and meaningful. First, in view of some inequality techniques, several sufficient conditions are established to ensure the passivity of the complex dynamical network with fixed topology and multi-weights. Second, for the complex dynamical networks with switching topology and multi-weights, some passivity criteria are also presented. Third, by employing pinning control technique, passivity problem is also discussed for multi-weighted complex dynamical networks with fixed and switching topologies.

2. Preliminaries

2.1. Notations

$0 \leqslant X \in \mathbb{R}^{n\times n}$ ($0 \geqslant X \in \mathbb{R}^{n\times n}$) means that matrix X is symmetric and semi-positive (semi-negative) definite. $0 < X \in \mathbb{R}^{n\times n}$ ($0 > X \in \mathbb{R}^{n\times n}$) means that matrix X is symmetric and positive (negative) definite. $\otimes$ represents the Kronecker product.

2.2. Some useful definitions

Definition 2.1 (see [35]). A system with supply rate $\mathcal{S}(u, y)$ is called dissipative if there exists a nonnegative function W satisfying

$$\int_{t_1}^{t_2} \mathcal{S}(u(t), y(t))dt \geqslant W(t_2) - W(t_1)$$

for any $t_1, t_2 \in [0, +\infty)$ and $t_2 \geq t_1$, where $u(t) \in \mathbb{R}^p$ and $y(t) \in \mathbb{R}^q$ are input and output of the system, respectively.

Definition 2.2. A system is passive if it is dissipative with regard to

$$\mathcal{S}(u(t), y(t)) = y^T(t)Fu(t)$$

where $F \in \mathbb{R}^{q\times p}$ is a constant matrix.

Definition 2.3. A system is strictly passive if it is a dissipative with regard to

$$\mathcal{S}(u(t), y(t)) = y^T(t)Fu(t) - u^T(t)Q_1u(t) - y^T(t)Q_2y(t)$$

for $\lambda_m(Q_1) + \lambda_m(Q_2) > 0$, $F \in \mathbb{R}^{q\times p}$, $0 \leqslant Q_1 \in \mathbb{R}^{p\times p}$ and $0 \leqslant Q_2 \in \mathbb{R}^{q\times q}$.

The system is input-strictly passive if $Q_1 > 0$ and output-strictly passive if $Q_2 > 0$.

3. Passivity analysis of multi-weighted complex dynamical networks with fixed and switching topologies

3.1. Passivity analysis of multi-weighted complex dynamical networks with fixed topology

3.1.1. Network model

The following equation is the mathematical model of complex dynamical network with multi-weights:

$$\begin{aligned}\dot{x}_i(t) &= g(x_i(t)) + \sum_{r=1}^{\sigma}\sum_{j=1}^{N} a_r G_{ij}^r \Upsilon_r x_j(t) + Bu_i(t) \\ &\quad + \sum_{r=1}^{\sigma}\sum_{j=1}^{N} \tilde{a}_r \tilde{G}_{ij}^r \tilde{\Upsilon}_r x_j(t - \tau(t)),\end{aligned} \tag{1}$$

where $i = 1, 2, \ldots, N$, $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^T \in \mathbb{R}^n$ stands for the state vector of the ith node; a_r and $\tilde{a}_r$ are positive real numbers; $g(x_i(t)) = (g_1(x_{i1}(t)), g_2(x_{i2}(t)), \ldots, g_n(x_{in}(t)))^T$ is a continuously differentiable vector function; $u_i \in \mathbb{R}^p$ denotes the input vector; $B \in \mathbb{R}^{n\times p}$ is a constant matrix; $0 < \Upsilon_r \in \mathbb{R}^{n\times n}$ and $0 < \tilde{\Upsilon}_r \in \mathbb{R}^{n\times n}$ are the inner coupling matrices for the rth coupling form; $\tau(t)$ is the time-varying delay and satisfies $0 \leq \tau(t) \leq \tau$ and $\dot{\tau}(t) \leqslant \delta < 1$; $G^r = (G_{ij}^r)_{N\times N}$ and $\tilde{G}^r = (\tilde{G}_{ij}^r)_{N\times N}$ are the coupling configuration matrices denoting coupling weights in the rth coupling form, if there is a connection between node i and node j ($i \neq j$), then $G_{ij} = G_{ji} > 0$ and $\tilde{G}_{ij} = \tilde{G}_{ji} > 0$; otherwise, $G_{ij} = G_{ji} = 0$ and $\tilde{G}_{ij} = \tilde{G}_{ji} = 0$ and the diagonal elements of matrices G and $\tilde{G}$ are defined by

$$G_{ii} = -\sum_{\substack{j=1\\ j\neq i}}^{N} G_{ij},\ \ i = 1, 2, \ldots, N,$$

$$\tilde{G}_{ii} = -\sum_{\substack{j=1\\ j\neq i}}^{N} \tilde{G}_{ij},\ \ i = 1, 2, \ldots, N.$$

In order to derive our results, the following assumption is introduced:

(A1) $g(\cdot)$ satisfies the following inequality:

$$(\eta_1 - \eta_2)^T P[g(\eta_1) - g(\eta_2) - S(\eta_1 - \eta_2)] \leqslant -\beta(\eta_1 - \eta_2)^T(\eta_1 - \eta_2)$$

for some $0 < \beta \in \mathbb{R}$ and any $\eta_1, \eta_2 \in \mathbb{R}^n$, where $0 < P = \text{diag}(p_1, p_2, \ldots, p_n) \in \mathbb{R}^{n\times n}$, $S = \text{diag}(s_1, s_2, \ldots, s_n) \in \mathbb{R}^{n\times n}$.

Assume that $\phi = (\phi_1, \phi_2, \ldots, \phi_n)^T$ is an equilibrium point of isolated node of the network (1), then it satisfies

$$\dot{\phi} = g(\phi) = 0.$$

Define $e_i(t) = x_i(t) - \phi$, we then have

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